Multidepot pickup and delivery problems in multiple regions: a typology and integrated model

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Abstract

The rapid development experienced by the transportation industry in the past decades has led to many configurations of networks and therefore to an explosion of variants in transportation problems, motivating researchers to look at broader logistic problems, beyond the basic vehicle routing problems. This work introduces a new type of problem scenario combining various attributes: a pickup and delivery problem with multiple regions, multiple depots, and multiple transportation modes. We provide definitions, a literature review, and a step-by-step construction of the mathematical models from a simple and well-known scenario to the multiregion multidepot pickup and delivery problem (MR-MDPDP). For each step the relevant literature is examined. Furthermore, we suggest possible extensions for prospective research.

Keywords: multidepot; pickup and delivery problem; multiregion; multimodal

1. Motivation

When thinking about realistic transportation networks, four main features come to mind: multiple depots, multiple regions, paired pickup and deliveries, and multimodality. The first three are the core pillars of the problems we are examining. The literature so far has mostly looked at transportation problems within one single region, although in reality transportation between different regions is common practice since the beginning of modern globalization. Also, the so often made assumption that the routing problem can be simplified to a vehicle routing problem (VRP) might be justified from an academic point of view, but is rather oversimplified for practical applications. Moreover, almost all companies operating on a somewhat larger scale will have a network structure that contains at least two depots, especially if transportation takes place between two geographically separated regions. The multimodality aspect usually occurs by default by operating in different regions and by the need for a switch of transportation modes for the last-mile delivery.
The great advances in infrastructures, computation power, and solution algorithms have increased the possibilities for all carriers, especially when facing long distance services. Therefore, we believe that research in the transportation field should start to look at more complex problem structures, like the ones described in this work. This paper is aimed to serve as an introduction to this new kind of problems that have been hardly studied in their entirety in the literature. Furthermore, we want to ascertain which works consider at least part of the main building blocks. We are therefore specifically looking at multidepot pickup and delivery problems (PDPs) in multiple regions to provide a foundation for further research in this field.

This research will not only benefit large transportation companies, but also small carriers who are willing to form a collaborative network system to be able to compete with those big companies in the long distance transportation market. The paper presents the transition from single-region multidepot problems to multiregion PDPs, going through all the basic mathematical models and existing literature in each field. This aims to provide a better understanding of the underlying multiregion transportation problem. Our goal is to gradually introduce the building blocks of our problem to allow the reader a deeper understanding of it. They are based on multidepot PDPs.

The paper is structured as follows: in Section 2, we will give a short problem introduction including all necessary definitions. Section 3 gives an overview of the relevant literature. Sections 4 and 5 introduce in detail different variations of multidepot PDPs in one or several regions. We will present mathematical models for each problem variation and give a literature review over work already done in these areas including information about already used instances. In Section 6, we conclude our work and in Section 7, we examine possible extensions to the basic problem presented before.

2. Problem introduction

Before going into detail we need to provide some definitions:

We assume a two-dimensional plane. All nodes within this plane are represented by a complete graph \( G(V,E) \), where \( V \) represents a set of nodes and \( A \) represents a set of arcs connecting them. A single arc connects a node \( i \) to a node \( j \), with a transportation cost of \( d_{ij} \). This cost is usually represented by the distance between both end points of the arc.

Our definitions are based on the nomenclature provided by Parragh et al. (2008b). We define a region as a group of nodes. The nodes in different regions are geographically separated. A pickup node is a customer, located at some specific location, where goods have to be collected by a vehicle. A delivery node is a customer where goods have to be dropped off. In unpaired problems, the goods are homogeneous and some quantity is collected and dropped off. There is no need to deliver specific goods. We are either looking at unpaired or at paired pickup and delivery nodes. For unpaired nodes the goods collected or dropped off are either transported between customers or have the depot as their origin or destination. Paired nodes are called a request. A request can have its pickup and delivery node within the same region or in different regions. A request therefore consists of exactly one pickup and one delivery node with a specific quantity. The pickup node has to be served before the delivery node. Since the goods are considered heterogeneous, the delivery node expects exactly these goods and not any goods just like it. Two modes of transportation are considered. Short-haul vehicles are used within a region. They are of small capacity and are mainly used for the
Fig. 1. (a) One region with two depots where all nodes can be served by either depot. (b) Two regions with two depots each. The depots belonging to different regions are connected by long-haul lanes. The delivery node of a corresponding pickup can be in the same or in another region. Black nodes are pickup points, while white nodes are the paired delivery points.

last-mile delivery. Long-haul vehicles are used for transporting goods between regions. They allow the consolidation of goods and have a much bigger capacity than the short-haul vehicles. Long-haul vehicles can be slow, fast, cheap, or expensive in any combination. A depot is the place where a vehicle starts and ends their tours. A tour starts and ends at the same depot. Also, vehicles are assumed to stay at the depot when they are not being used.

Figure 1 shows two cases of regions. All figures in this work use the following representation: the squares are depots, the circles are nodes. An empty (white) circle is a pickup node, a full (black) circle is a delivery node. Routes fulfilled by a short-haul vehicle are depicted as dashed lines, long-haul routes are double lines.

In general, the multidepot multiregion version of the VRP is closely related to network design problems (including location routing problems; LRPs), two- or multiechelon VRPs, multimodal transportation, and VRP with intermediate facilities. All of these topics are closely related to real-world problems and have been studied in the literature. However, no clear problem definition or consistent typology exists. Our goal is to provide structure and formulations for this generalization of existing problems. For a literature review on VRPs with multiple depots, we refer the interested reader to the publication by Montoya-Torres et al. (2015). The works mentioned there build the basis for the problem we are examining, but they do not contain the core characteristics of examining solely PDPs or dealing with multiple regions. For a literature review on multimodal freight transportation planning, we refer the reader to the work by SteadieSeifi (2014). Multimodal problems often incorporate multiple depots, PDPs, or multiple regions, but almost never all three. The literature review by Montoya-Torres et al. (2015) gives a very good overview but does not integrate all building blocks.

3. Literature

Our goal was to find literature that fits exactly to our problem: multiple depot PDP with interdepot routes in multiple regions. However, to the best of our knowledge, the exact problem we are studying here has not been yet addressed in the existing literature. Therefore, we set the boundary at literature dealing with multidepot problems with an underlying pickup and delivery routing problem. We loosened this boundary if a paper dealt with other important characteristics (interdepot routes or multiple regions) even if it was “only” a vehicle routing problem. Therefore, we also included in
our literature multidepot VRPs if they seemed fitting and had an overall network structure that contained some features of the problem classes. Because of this, literature about the two-echelon routing problem and work dealing with multiple modes became relevant for this work. We want to give structure to a new problem class because there has already been some work done, but it seems uncoordinated and looked at from very different point of views. We collected the relevant work to show the potential of this problem. Most of the work mentioned here used real-life instances provided by a company that had a need for a solution. Therefore, the importance becomes evident.

Table 1 gives an overview on the included literature. The individual papers are presented in detail in their corresponding sections. The papers are sorted first by the section they appear in this work and second by year of publication.

The first block of columns shows the underlying routing problem. This refers to the way the routing is dealt with within a region. We divide the papers according to the following classification: VRPs (VRP), paired PDPs (paired P&D), PDPs with backhauls (backhauls), and PDPs with mixed backhauls (mixed backhauls). In the standard VRP, each customer node has a certain quantity. It does not matter if this quantity has to be delivered or picked up since it is the same type for all nodes. Please note, that we did include literature that was similar enough to our problem, even if it only dealt with the standard VRP. The paired PDP (see the definitions by Parragh et al. 2008b) contains for each transportation request a pickup and delivery node that belong to each other. The delivery can be done directly (within the same tour as the pickup) or be forced to go over a depot. The VRP with backhauls or mixed backhauls belongs to a different class of problems. Here, all goods that are delivered to a customer are loaded at the depot and all goods that are picked up at a customer are unloaded at the depot. There is no link between the pickup and delivery nodes (see the definitions by Parragh et al. 2008a). In the mixed backhaul problem, the pickup and delivery at the customers can be fulfilled in any sequence. However, the standard backhaul problem (also called the VRP with clustered backhauls) specifies that all deliveries have to occur before the first pickup.

The second block of columns shows constraints and other problem specifications. Only constraints that were mentioned in the relevant literature are included in the table, apart from the vehicle capacity constraint, which was excluded because it appeared in every paper. These are however not all possible constraints since many problem variations have not been studied yet. For further suggestions, we refer the reader to Section 7. The third block of columns shows whether the solution method applied was an exact method or a heuristic. Because of the difficult nature of the problems, it is not surprising that heuristics were used more often. Also, it shows the number of locations (nodes) for the biggest instances solved. If only the number of requests were given, the assumption is made that each request consists of two locations and this number was added to the table.

4. Single region problems

Single-region problems are the basic building block for our new problem class. According to the previous definition of region, single-region problems encompass the vast majority of families of VRPs studied so far. The main elements of this setting are a set of customers or nodes that are to be visited and a set of depots from and to which routes start and arrive. Many configurations and restrictions of this setting can be found in the literature, leading to a broad spectrum of problems. The most representative problems of this setting are the classical traveling salesman problem and
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VRPs. Both problems have one unique depot. In Laporte (1992), the VRP is described in detail and solution algorithms for the VRP are presented. For the purpose of this paper, we are interested in problems where the number of depots is greater than one. Routing problems with one depot can be extended to several depots, leading to the family of multidepot problems. One of the classic problems among them is the multidepot VRP, which is an extension of the aforementioned VRP. A formal description of this type of problem can be found in the works by Polacek et al. (2004) and Renaud et al. (1996). The main elements of these settings remain the same as for the single-depot problems, only the graph $G(V,E)$ is extended to include all depots. The many existent typologies of problems in this family arise from the diversification of factors such as the nature of customers, their relation, time constraints, and many others. In this work, we focus on PDPs. We present some of the more relevant variants of this problem within the single-region setting, along with their mathematical model, before extending them to the multiregion setting.

**Multidepot vehicle routing problems**

Dondo and Cerdá (2007) consider a heterogeneous fleet of vehicles in a multi-depot scenario with time windows. The vehicles differ in capacity and speed. The underlying routing problem is a capacitated vehicle routing problem. Although they mention both pickups and deliveries, only one option is chosen at a time. The proposed method is a region-based optimization including three phases. In the first phase, a set of cost effective regions is identified, while in a second phase, regions are assigned to vehicles and tours are sequenced using a region-based MILP formulation. The third phase orders nodes within regions and schedules arrival times for the vehicles at customer locations by solving an MILP model. They test the computational performance of their algorithm on instances from Solomon’s work found in 1987. In addition, new test instances have been generated. They have up to 100 nodes and can solve many problems up to optimality or near-optimality. Even though the work deals only with a simple vehicle routing problem, it is mentioned because of the region-based solution approach that makes it interesting for our topic.

Another application presented by Currie and Salhi (2003) is a full-load PDP with time windows. They have heterogeneous vehicles and products. In their problem, they consider the delivery requirements of a large construction company. Goods have to be transported from multiple depots to a large number of customers. Since the problem consists only of deliveries, it can be classified as a capacitated vehicle routing problem. The problem ignores vehicle load splits as they very rarely occur. Trips are made between locations, where the product is collected, and the depots. Each location is visited exactly once, as several requests for the same location are considered as a single request for a location. Loading and unloading times are assumed to be constant, irrespective of the product to be transported. A constant cleaning time for vehicles, as well as waiting times might be added to the routes. “Dayworks,” which are situations in which the vehicles remain at a location for the rest of the day after their arrival, are also included. External vehicles might be hired by the company if the fleet is not sufficient to perform all trips.

To solve the problem described in Currie and Salhi’s (2003) work, the authors present a first formulation as a 0–1 linear programming model (LP) followed by a hybrid algorithm that chooses between a greedy heuristic and one based on regret costs. The greedy heuristic uses an index of stringency to select the next request for insertion, where requests with the lowest index values are
assigned to vehicles first. Stringency includes different attributes such as quantity and length of time windows. The regret costs method calculates a penalty cost for each request, which is obtained from the cost of forfeiting the opportunity of inserting the request in its best slot. They randomly generate their instances and solve instance sizes up to 200 location sites and 500 request. These instances can be found at: http://www.mat.bham.ac.uk/S.Salhi/others.

4.1. Multidepot vehicle routing problem with mixed backhauls

The multidepot vehicle routing problem with mixed backhauls consists of a set of unpaired pickup and delivery customers. Unpaired nodes are independent of each other, there is no relationship between the pickup and the delivery locations. By definition this means that all pickup goods are transported to a depot, and all delivery goods have to be loaded in the vehicle at a depot.

The underlying routing problem has been defined in different ways in the literature. Parragh et al. (2008a) refer to the problem as VRP with Mixed Linehauls and Backhauls, while Salhi and Nagy (1999) name it multiple depot VRPs with Backhauling. Traditionally, the VRP with backhauls has been associated with PDPs where pickup and delivery customers are not paired and a node has positive or negative quantities. In the simplified version (with clustered backhauls), the delivery points must always be visited before the pickup of goods to avoid infeasibility concerning the vehicle capacity restriction. However, in this section we present the more general case where no precedence relations have to be satisfied, that is, pickup and delivery points can be visited in any order. Therefore, we want to differentiate from the pure backhauling problems by referring to the problem as multidepot VRP with mixed backhauls (MDVRPMB). Figure 2 gives a visual representation of the VRP where mixed backhauls can be found.

As described above, the main characteristics of this problem are that the nodes in graph $G(V, E)$ could be either pickup or delivery customers, and that they can be visited in any order. The objective is to visit all customers at minimum cost while satisfying the capacity restrictions of the vehicles. A set of vehicle is defined in advance, but it is assumed that this number is sufficient to fulfill all transportation requests. The mathematical model is based on the work by Montoya et al. (2015).
Sets and parameters

\( N \) = set of customers,
\( L \) = set of depots,
\( V \) = set of all nodes, \( N \cup L \),
\( K_l \) = set of vehicles in depot \( l \),
\( K \) = \( \bigcup_{l \in L} K_l \) = set of all vehicles,
\( q_i \) = quantity load for the node \( i \)
\[ \begin{align*}
  +q_i, & \quad \text{if it is a pickup customer}, \\
  -q_i, & \quad \text{if it is a delivery customer}.
\end{align*} \]
\( Q^{SH} \) = capacity of each vehicle,
\( c_{ij} \) = travel cost between nodes \( i \) and \( j \).

Decision variables

\( x_{ijk} = \begin{cases} 
  1, & \text{if vehicle } k \text{ travels directly from node } i \text{ to node } j, \\
  0, & \text{otherwise.}
\end{cases} \)
\( S_{ik} \) = loading amount of vehicle \( k \) after visiting customer \( i \).

For simplicity in the notation, copies of depot \( l \) representing start and end points of the routes are denoted as \( l^0 \) and \( l^1 \).

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} \\
\text{subject to} & \quad \sum_{i \in N} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in V, \quad (1) \\
& \quad \sum_{i \in V} \sum_{k \in K} x_{ijk} = \sum_{j \in V} \sum_{k \in K} x_{ijk} = 0 \quad \forall k \in K, h \in N, \quad (2) \\
& \quad \sum_{i \in V} \sum_{j \in V} x_{ilk} = \sum_{j \in V} x_{ijk} \leq 1 \quad \forall l \in L, k \in K_l, \quad (3) \\
& \quad S_{jk} = \sum_{i \in V} x_{ijk} \cdot (S_{ik} + q_i) \quad \forall j \in V, k \in K, \quad (4) \\
& \quad 0 \leq S_{ik} \leq Q^{SH} \quad \forall i \in V, k \in K, \quad (5) \\
& \quad \sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \quad \forall k \in K, S \subseteq N, |S| \geq 2, \quad (6) \\
& \quad x_{ijk} \in \{0, 1\} \quad \forall i, j \in V, k \in K, \quad (7) \\
& \quad S_{ik} \in \mathbb{N} \quad \forall i \in V, k \in K. \quad (8)
\end{align*}
\]
Equation (1) represents the objective function, which is to minimize costs. Constraint (2) ensures that each node is visited once; constraint (3) that the vehicle entering and leaving a node should be the same (route continuity). Each short-haul tour originates and terminates at the corresponding depot (4). Loading restrictions for backhauls (5). Constraint (5) makes the model nonlinear. For simplification purposes, linearization of the constraint is not included in the model but in the Appendix. Since both pickup and delivery nodes are visited in any order, it is not sufficient to sum up over all quantities within a route. After every visited node, the quantity of goods in the vehicle has to be within our bounds (6). Constraint (7) is used for sub tour elimination.

Related problems

Different types of this model (with certain variants and assumptions) have been worked on. For example, the MDVRP with backhauling was early proposed by Min et al. (1992). They do not consider mixed backhauls because they assume rear-loaded trucks for their problem where reshuffling of loads is tedious and not easily possible. Therefore, all deliveries are fulfilled before the first pickup is allowed. Both their vehicles and depots have a limit on capacity. For the depots this means that they are able to only serve a limited amount of loadings and unloadings per day. Their main decision problems are to determine the fleet size (the number of available vehicles is not limited), to allocate the vehicles to the depots, to allocate the customers to vehicles, and the routing of the vehicles.

Irnich (2000) looks at a kind of multidepot PDP. The author here considers multiple depots and heterogeneous vehicles. The goods are delivered to and from a hub, instead of the depots of the particular vehicles. This is because of the particularity that all pickup or delivery locations serve as the depots for the vehicles. We classified it as a problem with backhauls under the assumption that the hub is actually the depot and the nodes are just simple customers. However, it is clear that it does not quite fit since the hub does not correspond to the definition of a depot. Their main decision is the assignment of a request to a specific vehicle. The routing is far less important.

Nagy and Salhi (2005) look at a multidepot VRP with mixed backhauls and simultaneous pickup and deliveries where a customer can both receive and send goods at the same time. Although their work focuses mainly on single-depot problems, they manage to extend this approach to a multidepot scheme. Their vehicles are capacitated and they have a maximum route length constraint.

Methods and instances

Min et al. (1992) decompose their problem and use a three phase heuristic method. Its first phase aggregates customers (delivery nodes) and vendors (pickup nodes) to capacitated regions; in the second phase, customers and vendors are assigned to a depot and a route; the third phase designs the individual vehicle routes. Each phase takes the output of the previous phase as an input. The data and instances used were provided by the transportation division of a large U.S. company and altered to ensure confidentiality. Their data contained 161 potential nodes to visit. The input data can be found in the appendix of their published paper.
Irrich (2000) formulated their problem as a network model. The goal is to find a set of trips that fulfills all requests while minimizing costs. They use a set partitioning/covering model that implies a two-phase algorithm. The solution approach first enumerates relevant route–vehicle combinations; then a set $T$ of relevant trips for each route–vehicle combination are enumerated, checked for dominance, and, if dominated, eliminated from $T$. Finally, they solve a set covering problem with the corresponding remaining set $T$. The algorithm was implemented in a decision support system used by the Deutsche Post AG. They have up to 22 locations with 242 requests in total and up to six different vehicle types. They do not provide a comparison to exact solutions but only to weak lower bounds. However, compared to previously manually planned solutions, they were able to reduce cost by 15% on average. As far as we know, the exact data have not been made public.

The multidepot problems from Nagy and Salhi (2005) are solved by dividing the customer set into borderline and nonborderline customers. For the nonborderline customers, the assignment is straightforward, whereas the borderline customers need an explicit assignment to their nearest depot. A customer is borderline if it is situated roughly midway between two depots. For each depot, they find a weakly feasible solution and finally insert the borderline customers into the vehicle routes one at a time. Their instances have up to 249 customers and up to five depots. They also reimplemented other methods from the literature to compare against and show that their methods outperform the others in relation to solution quality, but not necessarily in relation to computational times.

### 4.2. Multidepot paired pickup and delivery problem

This problem represents the basic extension to a multidepot scheme of the classic PDP. In PDP, customers are not independent any more, but they are paired to form requests. Each request consists thus of two customers or locations, one where the goods have to be picked up and a second one where goods are to be delivered. This problem differs from the one presented in Section 4.1 because the order in which customers are visited is now constrained. No delivery customer can be visited before its matching pickup customer. For further information about PDP problems, we refer the interested reader to the work by Parragh et al. (2008b) and by Berbeglia et al. (2007). Despite the extensive research conducted on single-depot PDP and its variants, very little literature exists for the multidepot case (MDPDP). A visual representation of the VRP with paired P&D can be found in Fig. 3.

#### New sets and parameters

- $R = \text{set of requests, } = \{1, \ldots, n\}$,
- $N = \text{set of pickup and delivery points, } = \{1, \ldots, n, n+1, \ldots, 2n\}$.

#### New decision variables

- $t_{i,k} =$ fulfillment time at node $i$ on tour (vehicle) $k$. 

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Fig. 3. A single-region multidepot VRP with paired P&D. Here, the delivery has to be after the pickup within the same route and vehicle. The nodes belonging together share the same number.

The precedence relation between pickup and delivery customers is modeled by explicitly considering the visiting time as a decision variable. The pickup and delivery nodes have to be always within the same route, it is not possible to store a package at the depot:

\[
\sum_{j \in N} x_{ijk} - \sum_{j \in N} x_{i+n,j,k} = 0 \quad \forall i \in R, k \in K, \tag{9}
\]

\[
t_{ik} \leq t_{i+n,k} \quad \forall i \in \{1, \ldots, n\}, \forall k \in K, \tag{10}
\]

\[
t_{jk} = \sum_{i \in V} x_{ijk} \cdot (t_{ik} + c_{ij}) \quad \forall j \in V, k \in K, \tag{11}
\]

\[
t_{ik} \in \mathbb{Z} \quad \forall i \in V, k \in K. \tag{12}
\]

Constraint (9) ensures that both pickup and delivery locations are served by the same vehicle. The precedence constraints for P&D are included by (10). Constraint (11) ensures that the time continuity is given in a tour.

Related problems

Bettinelli et al. (2014) present a multidepot heterogeneous PDP with soft time windows. They find a set of routes of minimum length for a fleet of vehicles with limited capacity to serve a set of customers. Routes start and end at a depot from a given set of depots. Customers have a given demand that has to be served by a single vehicle from a heterogeneous fleet. The violations of the time windows are penalized. The pickup and delivery of a package belonging to one request has to be done by the same vehicle.

Detti et al. (2017) present a multidepot dial-a-ride problem with heterogenous vehicles. They extend the problem by introducing compatibility constraints. The authors study a problem in health care, where patients have to be transported in vehicles depending on their needs. A fleet
of heterogeneous vehicles are located in geographically distributed depots. Patients may ask to be transported by a specific vehicle, which is called patient’s preferences. The problem consists of assigning transportation services to vehicles and finding a route for them. Among the constraints included in this problem, we can find vehicle’s capacity, patient–vehicle compatibility, pickup and delivery time windows, patients’ preferences, precedence constraints, quality, and timing of the service provided.

Methods and instances

As for the problem described by Bettinelli et al. (2014), the authors propose a branch-and-cut-and-price algorithm. First, a set of columns is generated for every customer, representing optimal paths when customers are served one at a time. A dummy column with a very high cost is included to ensure feasibility at each node of the search tree. They solve the linear relaxation of the restricted master problem (RMP), and then search for columns that are not in the RMP, but have negative reduced cost. If this column does not exist, the solution is optimal for the linear relaxation of the master problem and provides a lower bound to the problem. Their next step is a dynamic programming heuristic to provide an upper bound on the amount of dual prizes that could be collected when completing the path. To strengthen the lower bound they add violated two-path inequalities. The branching policies used are branching on the number of vehicles and branching on arcs. This approach is tested on instances derived from Solomon (1987), solving instances up to 144 customers. They manage to prove optimality for instances up to 75 customers.

In Detti et al. (2017), the proposed solution methods include variable neighborhood search and tabu search algorithms. A mixed integer linear programming formulation is also presented. These algorithms are compared based on their computational results for large real world and random instances. For the tabu search, in a first phase, an initial solution is generated through a fast insertion heuristic. Then, a tabu search scheme iteratively tries to improve this solution. At each iteration, a neighborhood is generated by perturbing the current solution. The whole process is repeated until a stopping criterion is satisfied. As for the variable neighborhood search algorithm, two steps are proposed. First, an initial solution is generated through a fast insertion heuristic. Then, a local search step is performed on this initial solution. The second step of the algorithm is an iterative procedure. At each iteration of the algorithm, a new randomly generated solution is created in the current neighborhood. A local search step is applied to this solution. If it is feasible and the best so far, it replaces the initial solution. If only one violation on any single type of constraint occurs, the solution is changed by an adjusting procedure to make it feasible. These methods are tested on random instances based on real-life data. The instances have up to 246 transportation services (requests) and 313 vehicles with four different types, distributed over 17 depots and belonging to 29 nonprofit organizations.

4.3. Multidepot pickup and delivery problem with interdepot routes

Here, we present one variant of the multidepot problem where depots are connected by interdepot lanes. Figure 4 shows a simple version of an interdepot route connecting two depots in a single
regions with paired pickup and delivery nodes. This setting makes sense when the vehicle type used for interdepot transportation is different from the one used for servicing the customer nodes, exploiting the advantages of a faster or cheaper connection. Therefore, this problem lies also in the family of multimodal transportation. The interdepot lane can be a permanent connection or opened when needed. The connection allows the consolidation of goods and a more efficient and maybe even faster transportation per unit of distance. It can be operated on a fixed schedule and would therefore mimic a third party providing the service, or on a flexible schedule which would mimic long-haul vehicles at the carriers’ own disposition. The costs can be fixed (full-truckload transportation) or variable depending on the goods transported. To represent different modes of transportation, multiple parallel lanes can be added to a model, for example, a cheap and slow lane for transportation by ship and an expensive and fast lane for transportation by plane. To the best of our knowledge, this problem has not been previously addressed in this setting in literature. However, it is included in this work as a basis for the development of multiregion transportation problems presented in Section 5.

This mathematical model inherits the main characteristics of the previous models without interdepot routes, and adds interdepot-related sets, variables, and constraints. Also, decision variables regarding the beginning and ending times of the routes have to be defined to model the time consistency between the short- and long-haul vehicles.

New sets and parameters

\begin{align*}
H_l &= \text{set of interdepot vehicles in depot } l \in L, \\
H &= \text{set of inter-depot vehicles, } \bigcup_{l \in L} H_l, \\
Q_{LH} &= \text{capacity of each inter-depot vehicle}, \\
c_{lp} &= \text{cost of traveling between depots } l \text{ and } p \in L.
\end{align*}

New decision variables

\begin{align*}
y_{ilph} &= \begin{cases} 
1, & \text{if request } i \text{ is transported from depot } l \text{ to depot } p \text{ with vehicle } h, \\
0, & \text{otherwise.}
\end{cases}
\end{align*}
\[ z_{lh} = \begin{cases} 1, & \text{if interdepot vehicle } h \text{ is used for a transport between depots } l \text{ and } p, \\ 0, & \text{otherwise}. \end{cases} \]

\[ m_h = \text{starting time of vehicle } h, \quad h \in H, \]

\[ b_k = \text{beginning time of tour } k \in K, \]

\[ e_k = \text{ending time of tour } k \in K. \]

The adjusted objective function includes the costs of transporting between depots, which may be different from the costs of the vehicles used for the routes:

\[
\text{minimize } \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{l \in L} \sum_{p \in L} \sum_{h \in H_l} c_{lp} z_{lp}. \tag{13}
\]

Constraints to add:

\[
\sum_{j \in N \cup \{l\}} \sum_{k \in K_l} x_{ijk} e_k \leq \sum_{h \in H_l} y_{ilh} m_h + M \left(1 - \sum_{j \in N \cup \{p\}} \sum_{k' \in K_p} x_{(i+n)jk'}\right) \quad \forall i \in R, \quad l, \quad p \in L, \tag{14}
\]

\[
\sum_{j \in N \cup \{l\}} \sum_{k' \in K_p} x_{(i+n)jk'} b_{k'} \geq \sum_{l \in L \setminus \{p\}} \sum_{h \in H_l} y_{ilh} \left(m_h + d_{ip}\right) \quad \forall i \in R, \quad p \in L, \tag{15}
\]

\[
\sum_{i \in R} \sum_{h \in H_l} y_{ilh} q_l \leq Q^{LH} \quad \forall l, \quad p \in L, \tag{16}
\]

\[
\sum_{i \in R} y_{ilh} \leq M \cdot z_{lp} \quad \forall l, \quad p \in L, \quad h \in H_l, \tag{17}
\]

\[
y_{ilh}, \quad z_{lp}, \quad \forall i \in V, \quad l, \quad p \in L, \quad z_{lp} \in \{0, 1\} \tag{18}
\]

\[ m_h \in \mathbb{Z} \quad \forall h \in H. \]

Starting time of the interdepot vehicle transporting goods of request \( i \) should be greater than the arriving time of the short-haul route serving the pickup customer corresponding to the request (14). The big M component is necessary to allow for requests being transported directly from pickup to delivery, without going through the interdepot route. Starting time of the short-haul route delivering goods of request \( i \) should be greater than the arriving time of the long-haul route transporting the goods of the request (15). Constraint (16), for the quantity on the interdepot vehicle. Constraint (17) ensures that requests are only assigned to long-haul transports that are performed.

Constraints (14) and (15) make the model nonlinear and therefore intractable for a commercial solver. However they can be linearized, as shown in the Appendix. In spite of this, the model remains NP-hard and can only be solved to optimality for small instances.

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Related problems

As far as we know, this type of multidepot problem has not been yet addressed in the literature in this form. Its main feature is the existence of two different ways of transportation in two stages, which requires the combination of a scheduling and a routing problem. Extensive literature exists for similar problems, where two dependent decisions are to be made. For example, two-echelon vehicle routing problems (2E-VRP) and LRPs are frequently studied in the literature. Also another concept of MDPDP with interdepot routes has been addressed in the literature, where interdepot routes are understood as a vehicle starting and ending the route in a different depot, thus staying away from multimodal transportation. A detailed explanation is given in Section 4.3.1.

Gendron and Semet (2009) consider a multiechelon LRP for a fast delivery service, with three levels of intershipment facilities (hubs, depots, satellites). From satellites, the products are sorted and delivered to the customers. Transportation between levels is done in different transportation modes.

In Crainic et al. (2011), Perboli and Tadei (2010), Perboli et al. (2011), and Breunig et al. (2016), a 2E-VRP is presented. They define one central hub, a set of intermediate depots called satellites, and a set of customers with demands. The freight stored at the central hub must transit through the satellites and then be delivered to the customers. Two different types of vehicles are considered according to the level they serve. Neither route size nor number of visited customers are limited.

In Hemmelmayr et al. (2012), the 2E-VRP is also considered. Satellites locations are assumed to be known and a limit on the number of vehicles for both levels is considered. They are also able to solve an LRP with their approach by transforming the 2E-VRP to an LRP. Capacity limit and opening costs are considered for the depots.

Ghilas et al. (2016a, 2016b) use a very similar type of interdepot routes in their work. Their interdepot route is a scheduled active public transportation line traveling in the city of Amsterdam transporting both passengers and packages. All vehicles from a depot can serve all customer nodes without restrictions, but they have the possibility to shorten their routes by consigning the packages to a public scheduled line. At the end of the line, the packages have to be picked up by another vehicle belonging to another depot and delivered to their destination. They prove that introducing scheduled lines leads to a decrease in transportation cost compared to the solutions obtained without them.

Methods and instances

Gendron and Semet (2009) present both an arc- and a path-based formulation for tackling the problem. Furthermore, they develop a binary relaxation as an alternative for which both formulations are equivalent. The computational results are obtained from 32 instances generated based on real data, for 93 depots, 320 satellites, and 700 customers. The linear programming relaxations are solved using CPLEX with a two hours CPU time limit.

For the 2E-VRP solution, Perboli and Tadei (2010) present new inequalities in an exact model. Inequalities are then inserted in a Branch and Cut framework with a 10,000 seconds computing time limit for each instance.
Crainic et al. (2011) present heuristics to solve the 2E-VRP. An initial solution is obtained by a first-clustering second-routing strategy. Then, a local search is performed over the clustering of customers around satellites. A final perturbation move is performed until a feasible solution is reached.

Perboli et al. (2011) present two interesting math-based heuristics to solve the 2E-VRP. The first one forces assignment variables to be fixed to a value and, if the model is infeasible, they unfix some variables and restart the process. The second heuristic method, solves a semicontinuous 2E-VRP using an MIP solver with a 60 seconds time limit and obtains a list of best integer solutions found. For every solution in the list, they consider the customer–satellite assignments and solve the VRP instances with a time limit of five seconds.

Crainic et al. (2011), Perboli and Tadei (2010), and Perboli et al. (2011) use instances from some previous work of theirs, which covers up to 50 customers and five satellites.

An adaptive large neighborhood search is proposed in Hammelmayr et al. (2012). Its main feature is that it allows for infeasibilities in the initial solution construction phase. Destroy and repair algorithms are used for improving the solution and making it feasible. The authors use the same instances as Crainic et al. (2011) and Perboli et al. (2011).

Breunig et al. (2016) make use of a large neighborhood search heuristic. The method uses several destroy operators and only one repair operator. They also work on collecting the instances for the problem, obtaining different sets from several previous works and modifying them to a common structure. Instances with up to 200 customers are solved. Results show a very good performance in comparison to the results from Hemmelmayr et al. (2012).

Ghilas et al. (2016a, 2016b) use both exact and heuristic methods for their problem. They decided for a branch-and-price approach as an exact method and for an adaptive large neighborhood search heuristic method. They managed to obtain exact results for small and medium size instances with up to 50 requests. In addition, they studied the effects of the number of scheduled lines, the scheduled lines frequency, and of the time windows on the operating costs. Their instances contain up to 100 requests and three scheduled lines.

4.3.1. Interpretations of inter-depot routes

So far, existing literature has referred to the interdepot routes as routes in which vehicles start and ends in different depots or when another depot than the starting depot is used as a replenishment or unloading point along the route. This interpretation leaves us with a quite different problem than the previously proposed one. Multimodal transportation stays out of scope and it is no longer a two-level decision problem.

The first option mentioned consist of vehicles starting in a depot and ending at a different location (depot or customer) within the same region. A practical application would be to allow the vehicles to return to their start depot on another day. Figure 5 shows a simple route with two pickup and
two delivery nodes that starts and ends at different depots. From this example, it is clear that by
forcing a vehicle to return to the start depot, a considerable increase in distance has to be condoned.

The paper by Goel et al. (2008) deals with a rich VRP that incorporates several constraints found
in real-life applications. These locations can require a pickup, a delivery, or simply a service, where
no quantity of goods change hands. They do not assume that vehicles are stationed at a depot,
but simply become available at a specific location, to a specific time with some predefined load.
Also, vehicles do not return to a depot, but an end location for the tour is given. Therefore, we
classified the problem as one with interdepot routes, since every location can be a depot. During
the computation, some requests get canceled while others became known only then. In addition
to the interdepot routes, they consider several constraints like time window restrictions or drivers’
working hours, among others.

They make use of a reduced large neighborhood search strategy, with several and specific neighbor-
hoods that allow them to go from one solution to another. They solved self-generated instances
with up to 500 vehicles and 2500 orders.

Another approach that can be considered as a problem with interdepot routes consists in allowing
a vehicle to visit another depot during its tour while still returning to its own depot at the end of the
day. This approach makes sense for unpaired PDPs or capacitated VRP with backhauls. Especially
vehicles with a tight capacity might profit from the possibility to unload or replenish the goods
to be transported during their route. This scenario also applies, for example, to electric vehicles
that need to use recharging stations during their route, as stated in the work by Hiermann et al.
(2016). Figure 6 shows an example of a route in which a vehicle visits a depot during its route while
transporting backhauls.

This problem has been introduced by Crevier et al. (2007) where depots act as replenishment facil-
ities, in a real-life grocery distribution network. Vehicles can have a single-depot route, which starts
and ends at the same depot, or interdepot routes, which connects two different depots. Nevertheless,
returning to its original depot is mandatory. They tackle the problem with a three-phase heuristic:
route generation, determination of least cost feasible rotations, and postoptimization phase. They
adapted instances from the literature and generated benchmark instances with 48–288 customers
and three to six depots.

Muter et al. (2014) tackle the same problem and allow vehicles to stop at intermediate depots for
replenishment. They solve their problem by a branch-and-price algorithm. Moreover, two different
pricing subproblems are implemented and compared. The traditional one-level column generation
scheme is inferior to their developed two-level decomposition scheme. They work on instances from the literature containing between three and seven depots and up to 50 customers, solving some of them to optimality. They show that allowing interdepot routes leads to an improvement of cost, although making the problem harder to solve.

Bard et al. (1998) and Angelelli and Speranza (2002) allow the replenishment at intermediate facilities, but differing from the previous works in that depots cannot act as intermediate facilities, hence being different kind of locations.

Angelelli and Speranza (2002) propose a tabu search algorithm to iteratively move from one solution to another within the same neighborhood. A new solution is chosen minimizing a penalty that is related to the objective function. Instances from previous literature are solved, as well as random generated ones with up to 150 customers, four satellites, and five vehicles.

The solution method proposed in Bard et al. (1998) starts by relaxing the vehicle routing problem with satellite facilities linear program. Subsequently, a branch and cut approach is implemented. They solve self-generated instances with up to 20 customers and two satellites.

5. Multiregion pickup and delivery problems

In this section, we introduce a new type of problems, the multiregion PDPs. The main characteristic of these problems is that regions are independent of each other, meaning that customers in different regions cannot be visited by a vehicle from a depot situated in another region. Especially, we are interested in describing the multidepot version of these problems (MR-MDPDP), where more than one depot is located in each region. This problem constitutes a special case of the MDPDP with interdepot routes explained in Section 4.3. From the application point of view, we think it seems to be more realistic than the MDPDP with interdepot routes since there exist many cases where transportation between two separated regions is unpractical for the vehicles performing the short distance transportation, and no other option than multimodal transportation is left.

In MR-MDPDP, all transportation requests have a pickup and a delivery node. These nodes can be within the same region (intraregion requests) or they can be in different regions (interregion requests). Customer locations (either pickup or delivery) are visited by short-haul vehicles. The goods from interregion requests have to be transported from the region where the pickup customer is located to the delivery customer region. This interregion transportation is done by long-haul vehicles. Long-haul or interregion vehicles connect the regions and transport the consolidated goods. They have to be scheduled so that all requests are fulfilled and cost minimized. In the case where there is only one depot in each region, the interregion vehicles represent a standing connection between the regions that have to be used due to lack of alternatives. However, the departure times can still be either fixed (scheduled) or flexible (up to optimization). If multiple depots are present in each region, the additional decision occurs, which of the possible long-haul connections to use. Since each depot can be connected to every other depot of any other region, several possibilities arise. Figure 7 shows a simple representation of a solution of a MR-MDPDP with two regions and two depots in each region.

To the best of our knowledge, this problem has not been considered in the literature yet. However, all related problems and literature detailed for the MDPDP with interdepot routes in Section 4.3 apply to the MR-MDPDP, and even in a greater degree, since in this case multimodal transportation
Fig. 7. MR-MDPDP with two regions and two depots per region. Requests A, B, and C are transported over long-haul lanes. Request D is an intraregion request served only by depot 2. The SH routes are numbered in their order of execution. The order is the following: Route 1 picks up request A, which then travels from D3 to D1 over the LH. Route 2 delivers A and picks up B. B travels over the LH from D1 to D4. Route 3 picks up D and C and immediately delivers D within the same route. Request C travels over the LH to D4. After both B and C have arrived in D4 they are both delivered by route 4.

is required and we face a pure two-level decision problem like the problems from 2E-VRP and LRP families.

The characteristics of this problem induce the need of introducing an index for the region in most of the sets and decision variables in the mathematical model. We also have to make a distinction between requests going over interregion transportation and requests for which pickup and delivery are in the same region. Since the introduction of this problem is one of the main goals of this paper and due to the several changes that need to be made on the mathematical formulation, we present the complete model for the multiregion problems.

Sets

\[ U = \text{set of regions}, \]
\[ R^I = \text{set of interregion requests, with customers located in different regions}, \]
\[ R^u = \text{set of intraregion requests, with both customers in region } u, \]
\[ N^u = \text{customer points in region } u, \]
\[ L^u = \text{set of depots in region } u, \]
\[ V^ u = N^u \cup L^u, \]
\[ K^u = \bigcup_{j \in L^u} K^u_j, \]
\[ H^u = \bigcup_{j \in L^w} H^u_j. \]

Parameters

\[ c_{ij}^u = \text{distance between points } i \text{ and } j \text{ in region } u, \]
\[ c_{lp}^w = \text{distance between depots } l \in L^u \text{ and } p \in L^w, \]
\[ d_{i}^u = \begin{cases} 1, & \text{if request } i \text{ starts in region } u, \\ 0, & \text{otherwise.} \end{cases} \]
$q_i^u$ = quantity to serve at customer $i$ in region $u$  
\begin{align*}
q_i^u &= \begin{cases} 
> 0, & \text{if it is a pickup point,} \\
< 0, & \text{if it is a delivery point,} \\
0, & \text{if customer } i \text{ is not in region } u.
\end{cases}
\end{align*}

$Q_{SH}^u$ = capacity limitation for short-haul vehicles, 
$Q_{LH}^u$ = capacity limitation for long-haul vehicles.

**Decision variables**

\begin{align*}
x_{ijk}^u &= \begin{cases} 
1, & \text{arc } i - j \text{ in region } u \text{ is covered by vehicle } k \in K^u, \\
0, & \text{otherwise.}
\end{cases} \\
y_{dlph}^{lw} &= \begin{cases} 
1, & \text{request } i \in R_l^u \text{ transported from depot } l \in L^u \text{ to depot } p \in L^w \text{ in long-haul vehicle } h \in H^u_l, \\
0, & \text{otherwise.}
\end{cases}
\end{align*}

\begin{align*}
z_{lw}^{ph} &= \begin{cases} 
1, & \text{vehicle } h \in H^u_l \text{ is used for transporting between depots } l \in L^u \text{ and } p \in L^w, \\
0, & \text{otherwise.}
\end{cases} \\
S_{ik}^u &= \text{quantity loaded in tour } k \in K^u \text{ after visiting customer } i, \\
t_{ik}^u &= \text{fulfillment time at customer } i \text{ from region } u \text{ with vehicle } k \in K^u, \\
b_{ik}^u &= \text{beginning time of route performed by vehicle } k \in K^u, \\
e_{ik}^u &= \text{ending time of route performed by vehicle } k \in K^u, \\
m_{ik}^u &= \text{starting time of interregion trip with vehicle } h \in H^u.
\end{align*}

For simplicity in the notation, copies of depot $l \in L^u$ representing starting and ending depot, respectively, in each region will be denoted as $l_{0}^u$ and $l_{1}^u$. Indexes in $N^u$ are constituted first with the customers belonging to inter-region requests and then with pickup and delivery customers belonging to intraregion requests in $u (R^u)$.

\[
\text{minimize} \quad \sum_{u \in U} \sum_{i \in V^u} \sum_{k \in K^u} c_{ijk}^u \cdot x_{ijk}^u + \sum_{u \in U} \sum_{w \in \overline{U}} \sum_{l \in L^u} \sum_{p \in L^w} \sum_{h \in H^u_l} c_{lp}^{lw} \cdot z_{lp}^{lw}.
\]  

\[
(19)
\]

Subject to:

\[
\sum_{j \in V^u} \sum_{k \in K^u} x_{ijk}^u = 1 \quad \forall u \in U, i \in N^u, \tag{20}
\]

\[
\sum_{i \in V^u} x_{ik}^u - \sum_{i \in V^u} x_{ik}^u = 0 \quad \forall u \in U, h \in N^u, k \in K^u, \tag{21}
\]

\[
\sum_{i \in V^u} x_{ik}^{u_{l_{0}^u}i} = \sum_{i \in V^u} x_{ik}^{u_{l_{1}^u},k} \leq 1 \quad \forall u \in U, l \in L^u, k \in K^u, \tag{22}
\]
\[ t_{jk}^u = \sum_{i \in V} x_{ijk}^u \cdot (t_{ik}^u + c_{ij}^u) \quad \forall u \in U, j \in V, k \in K, \]  

(23)

\[ b_k^u = t_{i,k}^u \quad \forall u \in U, k \in K^u, l \in L^u, \]  

(24)

\[ t_{i,k}^u < t_{i+|R^u|,k}^u \quad \forall u \in U, i \in R^u, k \in K^u, \]  

(25)

\[ S_{jk}^u = \sum_{i \in V^u} x_{ijk}^u \cdot (S_{ik}^u + q_i^u) \quad \forall u \in U, j \in N^u, k \in K^u, \]  

(26)

\[ 0 \leq S_{ik}^u \leq Q^SH \quad \forall u \in U, i \in V^u, k \in K^u, \]  

(27)

\[ \sum_{i \in R^l} y_{ilph}^{lv} \leq M \cdot z_{lph}^{lv} \quad \forall u \in U, w \in U \setminus u, l \in L^u \]  

(28)

\[ d_i^u \cdot \sum_{j \in V^u} \sum_{k \in K^u} x_{ijk}^u e_k^u \leq \sum_{h \in H^u} \sum_{p \in L^u} y_{ilph}^{lv} m_h^u \quad \forall u \in U, w \in U \setminus u, i \in R^l, l \in L^u, \]  

(29)

\[ \sum_{j \in V^u} \sum_{k \in K^u} x_{ijk}^w b_k^w \geq d_i^u \cdot \sum_{h \in H^u} \sum_{p \in L^u} y_{ilph}^{lv} (m_h^u + c_{lp}^w) \quad \forall u \in U, w \in U \setminus u, i \in R^l, \]  

(30)

\[ \sum_{i \in N^w} y_{ilph}^{lv} q_i^w \leq Q^{LH} \quad \forall u \in U, w \in U \setminus u, l \in L^u \]  

(31)

\[ x_{ijk}^u \in \{0, 1\} \quad \forall i, j \in V, k \in K, \]  

(32)

\[ y_{ilph}^{lv}, z_{lph}^{lv} \in \{0, 1\} \quad \forall i \in V, l, p \in L, h \in H, u, w \in U, \]  

\[ t_{ik}^u, S_{ik}^u, b_k^u, e_k^u \in \mathbb{Z} \quad \forall i \in V, k \in K, \]  

\[ m_h \in \mathbb{Z} \quad \forall h \in H. \]  

Constraint (20) ensures that all customers in region \( u \) are visited once and only once by a vehicle from region \( u \). A vehicle arriving at a customer must leave it in the same tour (21). A vehicle belonging to depot \( l \) in region \( u \) has to arrive at depot \( l \), if it leaves it (22). Constraint (23) refers to time continuity constraints: the arriving time to customer \( j \) with vehicle \( k \) equals the time the
previous customer of vehicle $k$ was serviced plus the travel time between $i$ and $j$. Constraints modeling the beginning and end time of vehicle $k$: Start time equals the time when vehicle $k$ leaves the depot. End time is when vehicle $k$ arrives at the depot (24). Time precedence constraint for intraregion pickup and delivery requests (25). Constraints (26) and (27) model the load of a short-haul vehicle when visiting customer $j$ and ensure the load stays below the short-haul capacity limitation. Constraint (28) binds decision variable $y$ and $z$. Constraints (29) and (30) bind decision variables $x$ and $y$. If interregion request $i$ starts in region $u$, the end time of the vehicle $k$ visiting the pickup customer must be smaller than the departing time of the long-haul vehicle $h$ transporting request $i$. Equivalently, the beginning time of vehicle $k'$ visiting the delivery customer must be greater than the arriving time of the long-haul vehicle $h$ to depot $p$. The load on a long-haul vehicle cannot exceed the long-haul vehicle capacity limitation (31).

The linearization of constraints (26), (29), and (30) is shown in the Appendix.

**Methods**

As previously stated, the MR-MDPDP is a two-level decision problem in which interrelated scheduling and routing decisions have to be made. Any change in the scheduling of requests traveling in long-haul vehicles affects the routes in both regions and vice versa. Therefore, we face a problem with two simpler underlying problems. The routing problem corresponds to a MDVRPMB (see Section 4.1). It is based on the classical capacitated VRP, which is itself an NP-hard problem (see Cordeau et al. 2002). Therefore, the MR-MDPDP also belongs to the category of NP-hard problems. This means that an exact approach to the solution is only possible for instances of very small size, and therefore a heuristic approach is needed for solving problems of a realistic size.

To assess how far we can get with the use of commercial solvers, the previous model has been implemented and applied to instances of different sizes. A time limit of 2 hours was set for every run. All computational results and data description can be found as online supplements to this publication. In general, within the specified computational time, we are able to solve instances with up to four interregion requests and four intraregion requests.

Consequently, heuristic algorithms are the only feasible option for dealing with bigger instances. Many types of algorithms have been used for problems with similar two-level decision structure, such as the families of 2E-VRP and LRP. Looking at the most relevant 2E-VRP literature (presented in Section 4.3), however, it is possible to observe that most of heuristics used consist of two-stage heuristics due to the fact that they provide convenient solution representation. The interrelated variables make it difficult to store a solution in a format required to use them as chromosomes in a population for genetic-based algorithms or as labels in a column generation scheme. Hence, large neighborhood heuristics arise as common practice to solve these problems, since the work is done mainly on one single solution. Also, most of the papers make use of two phase algorithms for obtaining initial solutions or to explore neighborhood spaces. That is, they separate the scheduling and the routing problems and consider one of them as an independent problem. This first problem is solved, and afterwards the second one is approached according to the solution obtained from the first phase. However, we found an interesting case in the work from Ghilas et al. (2016a), where a branch-and-price algorithm is successfully applied, getting solutions that outperform in quality those from the ALNS approach, and in time those from CPLEX. However, as earlier mentioned,
they need to design a complex labeling system, far from the standard labeling systems applied in general VRP problems.

Everything stated above suggests that probably the best way of approaching a solution method for the MR-MDPDP would be by making use of large neighborhood heuristics, as well as two-phase heuristics with a more particular design focused on each type of problem.

6. Conclusion

Multiregion problem scenarios are becoming more common every day. Different real-world applications include intraregion as well as interregion transportation, in which regions are connected by vehicles of larger capacity, different speed and cost.

In-town logistics are constantly evolving to reduce costs and increase productivity. While big trucks and trains are an advantage when traveling from one city to another, smaller trucks are used for pickup and delivery operations inside a city. Other multimodal models have to be used in certain cities, as the only transportation means that are allowed in some neighborhoods are small electric vehicles.

This work introduces these new type of logistic problems whose main focus lies on pickup and deliveries in multiple geographically separated regions. The characteristics of these problems lead to the necessity of dealing with other issues like multimodal transportation and multidepot networks. An exhaustive examination of already existing literature in related topics is also presented, justifying the multiregion scenarios as a logical evolution step toward more realistic problems in transportation logistics and setting a basis for further research in this direction. This work also contains a step-by-step construction of mathematical models, which highlights the relation between classical transportation problems and the new introduced logistic networks.

7. Outlook and possible extensions

As logistics operations are centralized to save resources, n-regions multimodal problems are an attractive scenario for further research. Companies can use large vehicles such as airplanes, trains, and trucks to connect main depots to a central location, smaller trucks can be used to reach smaller regional depots and from these, smaller depots, stores, dealers, and/or customers can be visited with small vehicles routes. It is very rare to find models that are capable of solving this kind of problems.

Multidepot, multiregion problems are of particular importance in collaborative settings. Although all cited studies assume to have one single decision maker who aims for a centralized solutions, this assumption does not necessarily hold for collaborative vehicle routing problems. Probably many companies are involved in such collaborative networks, and therefore decentralized decision making has to be investigated.

Collaborative networks have been studied and developed over the last years, in works like Berger and Bierwirth (2010), Gansterer and Hartl (2016), and Wang and Kopfer (2014). These studies were, however, focusing on single depot per carrier settings. In a multiple depot approach, collaborative networks could include collaborative depots that could be used by all participants. When thinking about a multiple region structure, collaboration might be even more beneficial, by allowing carriers...
to share depots and short-haul vehicles on every region, and by splitting costs in combined long-haul trips.

Real-world applications of collaborative networks can be found in Montoya et al. (2016) and Buijs et al. (2016), where a quantification of the obtained improvement suggests that collaboration is a plausible and recommendable practice. Once collaboration is performed, a final aspect to take into account is profit sharing. Guajardo and Rönqvist (2016) present a recent overview on cost and profit sharing allocation methods.

Besides solution methods, collaborative networks could be improved with managerial implications suggesting best practices in this field. The decision on how much information does a company need to share to get the maximum advantage is one of the main questions.

Furthermore, we introduce some possible extensions for the problems. All extensions are presented with respect to the model in Section 5. These show the extensive gap between already existing literature and a rough approximation to reality, making these problems an attractive field for future research since many real-world extensions are not considered so far.

7.1. Time windows

Time windows have already been used a lot in the literature and can be added in many different ways. Usually the nodes or customers to visit have soft or hard time windows for the visit. A soft time window that is not met results in penalties that impair the objective value. A hard time window that is not met results in an infeasible solution. Time windows can also be considered on the depot opening hours. A clear example of a routing problem with time windows can be found in Dondo and Cerda (2007).

Some changes should be done to the presented models when including time windows. Let \([a_i, f_i]\) be the time window for customer \(i\).

- Soft time windows. The objective function must be modified to account for penalty costs. Let \(\phi\) represent the relative penalty cost. The new objective cost would be:

\[
\text{minimize } \sum_{u \in U} \sum_{i, j \in V_u} \sum_{k \in K_u} c_{ij}^u \cdot x_{ijk}^u + \sum_{u \in U} \sum_{w \in \bar{u}} \sum_{l \in L_u^w} \sum_{p \in L_u^w} \sum_{h \in H_u^w} c_{lpw}^u \cdot z_{lpw}^u + \sum_{u \in U} \sum_{i \in V_u} \sum_{k \in K_u} \phi \cdot \max(0, t_{ik}^u - f_i). \tag{33}
\]

- Hard time windows. New constraints have to be added.

\[
a_i \leq \sum_{k \in K_u} t_{ik}^u \leq f_i \quad \forall u \in U, i \in V_u. \tag{34}
\]

7.2. Multiple periods

Another common extension in routing problems is to consider multiple periods. Generally, these problems introduce the possibility for requests to be performed on different days. That induces
another degree of freedom to the combinatorial problem, widening the solution space. Servicing a
customer on different days can lead to significant improvements in the objective function. Storage
of goods in depots is a realistic option for these problems.

The work of Vidal et al. (2012) contains a clear explanation of multiple period models. A hybrid
genetic metaheuristic is developed to solve a variety of problems within the multidepot and periodic
VRP families.

Multiple periods induce a change on the variables of the model. A temporal index should be
added to all variables that could be dependent on it. If we name \( o \) the new index representing
the days in the planning horizon, the variables of our model would be the following:

\[
x_{ijko}^u, y_{ilpho}^{mow}, z_{lpho}^{mow}, s_{iko}^u, t_{iko}^u, b_{ko}^u, e_{ko}^u, m_{ho}^u.
\]

7.3. Heterogeneous vehicles

A problem with an heterogeneous fleet is a problem where vehicles with different problem-relevant
characteristics exist. It refers to vehicles within the same mode of transportation and performing
the same kind of service. Vehicles can be different if they have different capacities, fixed or variable
cost, range (especially for electric vehicles), driving time restrictions or restrictions concerning the
arcs they can travel, different speed, or different requirements for the drivers (not all drivers have a
license for all vehicle types). In a realistic scenario, a company will generally have an heterogeneous
fleet.

Regarding heterogeneous fleets, the following works have to be mentioned: The work by Dondo
and Cerdá (2007) is a multidepot VRP with heterogeneous fleet and time windows. Their vehicles
differ in capacity, cost, and speed. Irnich (2000) works on a single region multidepot VRP with
an heterogeneous fleet. The vehicles in the fleet differ by cost and speed (regarding driving and
loading), and capacity. Salhi and Sari (1997) consider a single region multidepot capacitated VRP
with the objective to construct vehicle routes and determine the composition of the vehicle fleet.
The vehicles in the fleet differ in capacity and cost. The aim of the authors is to construct a set of
routes and determine the vehicle fleet composition minimizing costs.

The changes on our model derived from the use of heterogeneous fleet would be reflected on the
parameters and sets. Also we would need an extra index for the variables. Assume we have \( n \) vehicle
types for the short-haul routes and \( r \) vehicle types for long-haul routes. The new elements would be
\( K_{ln}^u, K_n^u, H_l^u, H_r^u \), sets of short- and long-haul vehicles of each type in depot \( l \); \( c_{ijn}, c_{lpr}, Q_{SH}, Q_{LH} \),
costs of transportation and capacities for each vehicle type; \( x_{ijkn}^u, y_{ilphr}^{mow}, z_{lphr}^{mow}, s_{ikn}^u, t_{ikn}^u, b_{kn}^u, e_{kn}^u, m_{hr}^u \),
variables containing information about the vehicle type.

7.4. Multiattribute vehicle routing problems

Multiattribute or rich VRPs are labels for VRPs with additional constraints that aspire to express
a more realistic problem structure and decision choices (multiple depots, fleets, and commodities
that take place in multiple periods and have to consider driver work rules, traffic congestion, and
much more).

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A rich VRP can be found in the work by Ceselli et al. (2009), who use a heterogeneous fleet of vehicles; multiple depots; multiple capacities; time windows associated with depots and customers; incompatibility constraints between goods, depots, vehicles, and customers; maximum route length and duration; upper limits on the number of consecutive driving hours and compulsory drivers’ rest periods; the possibility of skipping some customers and using express courier services instead of the given fleet to fulfill some orders; the option of splitting up the orders; and the possibility of open routes that do not terminate at depots. Vidal et al. (2013) present a survey on heuristics for multiattribute VRPs. However, in spite of their extensive pool of constraints, both papers do not consider multimodality, the possibility of consolidation, or more complex network structures.

These considerations would lead to many different changes in our model from Section 5, depending on which attributes are added to the problem.

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**Appendix: Linearization of nonlinear constraints**

Some constraints in the previously presented models are not linear. Here, we present the linearization of these constraints to provide full information about the way the models can be implemented.

All nonlinear constraints can be grouped in two main equation classes.

1. The first one applies to time continuity and capacity constraints. They appear in constraints 11, 23, 5, and 26. An additional variable \( A_{ijk} \) is defined, and big \( M \) notation is used.

   Original constraint:
   
   \[
   S_{jk} = \sum_{i \in V} x_{ijk} \cdot (S_{ik} + q_i), \quad t_{jk} = \sum_{i \in V} x_{ijk} \cdot (t_{ik} + c_{ij}) \quad \forall j \in V, k \in K.
   \]

   New constraints:
   
   \[
   x_{ijk} + A_{ijk} = 1 \quad \forall i, j \in V, k \in K,
   \]
   
   \[
   S_{ik} + q_i - S_{jk} \leq MA_{ijk} \quad \forall i, j \in V, k \in K,
   \]
   
   \[
   S_{ik} + q_i - S_{jk} \geq -MA_{ijk} \quad \forall i, j \in V, k \in K.
   \]

2. The second linearization applies to the flow consistency constraints for the models with inter depot routes 14, 15, 29, and 30. Two new decision variables are defined: \( \alpha_{ik}, \beta_{ilh} \).

   Original basic constraint:
   
   \[
   \sum_{j \in N \cup \{l\}} \sum_{k \in K_i} x_{ijk} e_k \leq \sum_{h \in H_j} y_{ilph} m_h \quad \forall i \in R^l, l, p \in L.
   \]

   New constraints:
   
   \[
   \sum_{j \in N} x_{ijk} e_k = \alpha_{ik} \quad \forall i \in R^l, k \in K(nonlinear),
   \]
\[
\sum_{l' \in L \setminus l} y_{il'l} m_{h} = \beta_{ilh} \quad \forall i \in R^l, l \in L, h \in H^l \text{(nonlinear)},
\]

\[
\sum_{k \in K^l} \alpha_{ik} \leq \sum_{h \in H^l} \beta_{ilh} \quad \forall i \in R^l, l \in L.
\]

The first two of the new equations have to be linearized too. We show the linearization of the first one, and the same method applies to the second one. Big \( M \) notation is again used.

\[
\alpha_{ik} \leq M \sum_{j \in N} x_{ijk} \quad \forall i \in R^l, k \in K,
\]

\[
\alpha_{ik} \leq e_k \quad \forall i \in R^l, k \in K,
\]

\[
\alpha_{ik} \geq e_k - M (1 - \sum_{j \in N} x_{ijk}) \quad \forall i \in R^l, k \in K.
\]