MIGRATION OF STAR CLUSTERS AND NUCLEAR RINGS

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ABSTRACT

Star clusters that form in nuclear rings appear to be at slightly larger radii than the gas. We argue that the star clusters move out from the gas in which they are formed because of satellite-disk tidal interactions. In calculating the dynamics of this star cluster and gas ring system, we include the effects of dynamical friction of the background stars in the host galaxy on the star cluster, and inflowing gas along the bar onto the nuclear ring at the two contact points. We show that the final separation is of the order of the Hill radius of the nuclear ring, which is typically \(20 \sim 30\%\) of its radius. Massive star clusters can reach half of this separation very quickly and produce a factor of a few enhancement in the gas surface density. If this leads to star formation in addition to the (ongoing) formation of star clusters near the contact points, a possible (initial) azimuthal age gradient may become diluted or even disappear. Finally, if the star cluster are massive and/or numerous enough, we expect the nuclear ring to migrate inward, away from the (possibly) associated (inner) Lindblad resonance. We discuss how these predictions may be tested observationally.

Subject headings: accretion, accretion disks — stellar dynamics — stars: formation — galaxies: bulges — galaxies: star clusters — galaxies: nuclei

1. INTRODUCTION

Nuclear rings in barred spiral galaxies are regions of large gas surface densities and high star formation. Their existence is intimately related to the stellar bars with which they are associated. The stellar bar drives the interstellar medium (ISM) into the nuclear ring. How the radius of the ring is determined is unclear. The radius may be set by the location of the inner Lindblad resonance (ILR) of the stellar bar (see review by Buta & Combes 1996, and references therein). The reasoning behind this idea is that the sign of the torque changes on either side of the Lindblad resonance. Alternatively, the radius may be at the location where the non-axisymmetric bar weakens, which is typically at \(\sim 10\%\) of the size of the bar (Shlosman et al. 1990). Finally, it may also be determined by where the transition from phase space domination of \(X_1\) orbits (parallel to the major axis of the large-scale bar) to the phase space domination of \(X_2\) orbits (perpendicular to the bar) occurs (Regan et al. 1997).

Regardless of how they are formed they are interesting as a class because they are perhaps the largest population of nearby star bursting regions. They form star clusters in prodigious amounts and are the only environment where super star clusters (SSCs) might be found in abundance in normal galaxies (Maoz et al. 2001). These SSCs can have masses in excess of \(10^6\) \(M_{\odot}\) and may be the progenitors of globular clusters if they can survive for \(10^{10}\) years.

In some of these systems the location of these star clusters is curious. They appear to be at larger radii than the gas ring from which they are presumably formed. For instance, the morphology of NGC 1512 clearly show the star cluster(s) exterior (radially) to the gas as opposed to being embedded in the gas (Maoz et al. 2001). Taking another example, in NGC 4314 the star clusters are at larger radii than the gas in the nuclear ring (Benedict et al. 2002). Finally, Martini et al. (2003) find that in their sample of 123 galaxies, eight have strong nuclear rings and in each of the eight, star formation occurs outside of the dust ring. Two of their other galaxies have dust lanes immediately interior to what appears to be an older stellar ring.

There are two possible explanations for why these star clusters are external to the gas. First, while the gas can lose angular momentum due to dissipation, the stars and star clusters are collisionless and hence remain on larger orbits (Regan & Teuben 2003). Second, the star clusters migrated outward after their formation. We explore the latter possibility in this paper.

We argue that tidal interactions between the star cluster and the nuclear ring from which it was formed leads to their separation. We apply the physics of “gas shepherding”, which was studied by one of us (Chang 2008, hereafter C08), to this cluster-ring system. We derive a natural length scale for this separation, which we call the “Hill radius” of the ring. At the same time, we show that the outer edge of the ring can experience a surface density enhancement of a few, possibly leading to additional formation of stars and star clusters along the edge of the ring. Finally, we find that if the star clusters are massive and/or numerous enough the cluster-ring systems as a whole is expected to migrate inward. We show that these predictions can be tested observationally.

We present our model in §2. After summarizing the observed properties of nuclear rings in §2.1, we give the basic picture and introduce the Hill radius of the ring in §2.2 followed by the basic equations of our model in §2.3. The solutions for various scenarios are outlined in

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A few observationally testable predictions that arise from our model are given and compared to observations in § 4. We discuss some aspects of our model in § 5 and summarize our conclusions in § 6.

2. TIDAL INTERACTIONS BETWEEN STAR CLUSTERS AND NUCLEAR RINGS

We first summarize the basic properties of observed nuclear rings, which are used to place the model predictions in context. We then present the basic picture of our model, followed by the basic set of equations that govern our model.

2.1. Observed properties

For a sample of 22 nuclear rings, Mazzuca et al. (2008) find that the rings are in the same plane as the disk of their host galaxy and that they are intrinsically circular. Typically, the radius of the ring is around ∼0.5 kpc, but it can be a factor three smaller or larger. With a circular velocity at the ring radius of around ∼50 km s⁻¹, the (spherically) enclosed mass is about \( M_{\text{enc}} \sim 2 \times 10^5 M_\odot \).

The gaseous mass in the ring may be estimated from high spatial resolution CO measurements, using the 'standard' conversion to \( M_\odot \) and adding 36% for He. In this way, we obtain for NGC 5457, NGC 3351, NGC 6951, NGC 3504 (Kenney et al. 1992, 1993) and NGC 4314 (Benedict et al. 1996) on average a mass inside the ring of \( M_d \sim 5 \times 10^5 M_\odot \). Although the ring mass may be as much as a factor ten smaller or two larger, it correlates with the ring radius, so that the ring-to-enclosed mass ratio \( q_d \equiv M_d/M_{\text{enc}} \) stays within a narrow range of \( q_d = 0.01 - 0.03 \). While this is the molecular gas inside the ring, part of the gas is being ionized, due to photons from newly formed massive stars or due to shocks from stellar winds or supernova explosions, and at the inner edge of the ring possibly even due to an active galactic nucleus (AGN). However, the ionized gas mass estimated from the Hα (plus [N II]) emission is generally two orders of magnitude lower than the molecular gas mass (e.g. Planesas et al. 1997).

The nuclear rings are easily observable due to the emission from the massive stars inside the young star clusters in the ring (which in the ultraviolet even dominates the emission from a possible AGN). The combination of high-spatial resolution (HST) images and multi-wavelength coverage, makes it possible to isolate the individual clusters and after correction for (dust) extinction to estimate their mass and (relative) age. Below we use such a study by Benedict et al. (2002) to investigate in more detail the nuclear ring in NGC 4314. Overall, the cluster masses follow a power-law distribution with index −2, similar to young star clusters in merging galaxies such as the Antennae (Zhang & Fal§ 1999), but typically extending to a smaller upper limit in mass (few times \( 10^5 M_\odot \)), i.e., fewer of the so-called “super star clusters” (but see Maoz et al. 2001).

On the other hand, these young star clusters are most likely just the latest star formation in an ongoing succession of bursts (e.g. Allard et al. 2006). Most of the lower-mass clusters dissolve, either early-on (\( \lesssim 50 \) Myr) due to violent relaxation after expelling the residual gas (‘infant mortality’) depending on the star formation efficiency (e.g. Parmentier et al. 2008), or later-on due to internal relaxation (‘evaporation’) and external tidal effects (e.g. McLaughlin & Fall 2007). The resulting evolution towards a bell-shaped distribution implies a relative increase in more massive star clusters over time. Unfortunately, once the most massive stars have died, the luminosity of star clusters very rapidly fades, so that it becomes more difficult to distinguish them from the old stellar population of their host galaxy. Nevertheless, based on velocity dispersion measurements from stellar absorption lines, Hägele et al. (2007) estimate dynamical masses up to almost \( 10^7 M_\odot \) for individual star clusters in the nuclear ring of NGC 3351. Henceforth, the typical ratio of cluster-to-ring mass ratios \( q' \equiv M_s/M_d \) depends on the (average) age of the star clusters. Given a young, luminous star clusters we adopt the range \( q' = 0.001 - 0.01 \), whereas for the old(er), faint(er) star cluster \( q' = 0.01 - 0.1 \) is probably more appropriate. Below we investigate the two cases \( q' = 0.1 \) and \( q' = 0.01 \), corresponding to \( M_s \sim 5 \times 10^6 M_\odot \) and \( M_s \sim 5 \times 10^5 M_\odot \) in case of the above average gas mass in nuclear rings of \( M_g \sim 5 \times 10^4 M_\odot \). The effect of a less massive star cluster with \( q' < 0.01 \) is similar to that of \( q' = 0.01 \).

The time for a star cluster to cross the nuclear ring is given by \( t_{\text{cross}} = 2\pi/\Omega \), where \( \Omega = \sqrt{GM_{\text{enc}}/r^3} \) is the orbital frequency and \( G \) is Newton’s constant of gravity. Given a typical circular velocity of \( \sim 150 \) km s⁻¹ at the observed mean nuclear ring radius of 0.5 kpc, the crossing time is around \( t_{\text{cross}} \sim 20 \) Myr. A star cluster is subject to dynamical friction with the bulge stars on a (minimum) timescale\(^3 \) \( t_{\text{df}} = M_{\text{enc}}/(\Omega M_{\text{in}} \ln \Lambda) \), with \( \ln \Lambda \) the Coulomb logarithm. Using \( t_{\text{cross}} = t_{\text{df}} q_d' q_d \pi r^3 M_{\text{in}} \) and substituting \( q_d \simeq 0.02 \) from above, we find that the dynamical friction time in units of the crossing time is approximately given by \( t_{\text{df}}/t_{\text{cross}} \simeq 1/q' \). This means hundreds to tens of orbits for star clusters with masses that are 1% (\( q' = 0.01 \)) to 10% (\( q' = 0.1 \)) of the nuclear ring mass, or, with \( t_{\text{cross}} = 20 \) Myr, a dynamical friction time \( t_{\text{df}} \) from about 2 Gyr to 200 Myr for a non-rotating bulge, which we will assume for the remainder of this work.

2.2. Basic Picture

We illustrate our basic model in a cartoon in Figure 1. Gas flowing in a bar potential along the \( X_1 \) orbits transitions to \( X_2 \) orbits making up the nuclear ring. The \( X_2 \) orbits are approximately circular and so we will approximate them as circles. From the gas in the nuclear ring stars are formed in star clusters. We assume that the star clusters are formed preferentially near the outer edge. These newly formed and forming star clusters are subject to dynamical friction with the background bulge stars\(^4 \) and tidal interactions with the ring. However, the initial tidal interactions are strong compared to dynamical friction and thus these star clusters will initially move away from the ring.

Indeed, in observations of nuclear rings young star clusters often appear to be at larger radii than the

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\(^3\) The dynamical friction time is likely to be longer for a partially rotationally supported bulge as opposed to a non-rotating bulge.

\(^4\) The background bulge stars may also be halo stars, and the bulge/halo is assumed not to be supported by rotation. For the purposes of this work, we will idealize the bulge/halo as an isothermal background distribution, which we argue in § 5.1 is a valid approximation.
gas ring from which they are presumably formed. The color composite images of NGC 1512 (Maoz et al. 2001), NGC 4314 (Benedict et al. 2002), and NGC 7742 (Hubble Heritage5), show young star clusters as bright spots on the outside of gas traced by the dust. As pointed out by the referee, from these images it is also clear that, even though the nuclear rings are as we assume approximately circular with star clusters at the outer edge, the observations can be quite difficult and complex. First, when resolved in the optical the rings can look like tightly wound spirals. This might be just appearance because the bar dust lanes connecting with the ring are also partly obscuring the ring in optical broad-band images, whereas in e.g. narrow-band imaging around Hα the ring of young star clusters seem to be complete (see also Maoz et al. 2001). Nevertheless, our basic picture described above for a circular ring should still provide a fair description in case of a tightly wound spiral. Second, very obscured star clusters could remain hidden below the gas ring even in the near infrared images taken, whereas even longer wavelength observations (yet) lack the high spatial resolution required to resolve the star clusters. Even so, the clearing of gas and dust from the newly formed star cluster is very fast and efficient, as even the youngest visible star clusters are only mildly reddened (e.g. Maoz et al. 2001). Finally, some (sectors of) star clusters seem to appear more on the inside of the nuclear ring, possibly along an inward extension of the gas inspiraling onto the ring along the large-scale bar. As we discuss briefly in § 5.3, star clusters may form within the ring, in particular at the contact points where the gas flows into the ring. Moreover, the interaction of non-axisymmetric structures inside the ring (such as nuclear bars and spirals) with the interstellar medium may also induce the formation of star clusters. However, even though such additional mechanism might be operating and the observations should be interpret with care, they in general support our basic picture of a circular nuclear ring with young stars migrating outward.

Since the basic physics mirror the "gas shepherding" scenario studied by C08, we adopt the same notation. In this case the satellite is a star cluster and the disk is a ring, but since only the (local) geometry at the edge matters the physics remains the same. There are however two important differences with respect to C08. First, the perturbing star cluster’s initial radial position begins very close to the ring as opposed to a satellite reaching the disk from very far away in C08. Due to ring tidal interactions, the star cluster and ring will separate to establish an initial separation, Δr, which we show below is of order the “Hill radius” of the ring. We aim to study the manner by which this initial separation is achieved. Second, gas continues to flow in from the X1 orbits so that at the transition between the X1 to X2 orbits, gas is continuously being added to the system. Therefore, a source of mass exists in this system, which changes the dynamics.

For now we ignore the source of mass to concentrate just on the tidal interaction between the ring and star cluster and the dynamical friction experienced by the star cluster. We begin by showing that this combined action of dynamical friction and ring tides results in a length scale in the problem, which we call the Hill radius of the ring. The torque due to dynamical friction on the star cluster is (Chandrasekhar 1943; Binney & Tremaine 1987)

\[ T_{df} \sim \frac{M_s}{t_{df}} \Omega_s r_s^2 \sim \frac{GM_s^2}{r_s}, \]

where \( M_s \) is the mass of the star cluster, \( r_s \) is its distance from the center of the galaxy, \( \Omega_s \) is its orbital frequency, and, as before, \( t_{df} \) is the timescale for dynamical friction. The torque due to the excitation of spiral density waves in a disk is (Goldreich & Tremaine 1980; Lin & Papaloizou 1986; Ward & Hourigan 1989; Artymowicz 1993; Ward 1997)

\[ T_{disk} \sim \frac{GM_s^2}{r_s^3} \left( \frac{r_s}{\Delta r} \right)^{3}, \]

where \( M_d \) is the mass of the ring with radius \( r_d \). The radial separation between the star cluster and the ring \( \Delta r = r_s - r_d \) is assumed to be small compared to the distance to the center, i.e., \( \Delta r \ll r_s < r_d \). While dynamical friction torques down the star cluster, the ring torques up the star cluster. Balancing these two torques, i.e., \( T_{df} \sim T_{disk} \), gives

\[ \Delta r \sim \left( \frac{M_d}{M_{enc,s}} \right)^{1/3} = r_{H,d}, \]

where the latter defines the Hill radius of the ring.

Given the observed range in ring-to-enclosed mass of \( q_d = 0.01 - 0.03 \) (§ 2.4), the Hill radius is around 20—30% of the radius \( r_s \) of the star cluster. Since \( r_s \approx r_{H,d} \), the observed range in nuclear ring radii of \( r_d = 0.2 - 1.7 \) kpc (e.g. Mazzuca et al. 2008) results in \( r_{H,d} \approx 50 - 500 \) pc. We expect the radial separation between the star cluster and nuclear ring to be of order of this Hill radius. However, the exact amount and nature by which this separation will be achieved depends strongly on the strength of the tidal torque between the cluster and the ring, which we intend to quantify in this paper.

The second process which we wish to study is the effect of an additional source of mass on the dynamics of the cluster-ring system. The gas merges onto the nuclear ring at the two contact points where the X1 orbits transition to X2 orbits. We assume that the region where the gas is added to the nuclear ring is radially narrow compared to the radius of the nuclear ring. For a mass inflow rate of \( \dot{M} \), the amount of angular momentum added to a system per unit time is

\[ \dot{L} = \dot{M} \Omega r^2. \]

However, not all of this angular momentum can be immediately applied to affecting the qualitative dynamics of the system. By qualitative, we mean that the mass inflow rate, which acts as a source of angular momentum, changes the behavior of the cluster-ring system. For instance, the source of angular momentum from mass inflow balances the sink of angular momentum from dynamical friction on the star cluster. To estimate what mass inflow rate would be needed to effect such a change, we suppose this mass is added just inside of the satellite radial position, i.e., \( r \approx r_s \). The mass will then be pushed to the edge of the ring, \( r_d \), due to tidal interactions. The change in radial position will be of order the Hill radius

5 http://heritage.stsci.edu/1998/28/
of the disk, \( r_{H,d} \). Hence, for such a parcel of gas of mass \( M \), the change in angular momentum moving from \( r_s \) to \( r_d \) is

\[
\Delta L \sim M \Omega_r \Delta r \sim M \Omega_s r_s r_{H,d}.
\] (5)

For a constant mass inflow rate \( \dot{M} \), we expect the torque from this inflow of gas, acting on the star cluster, to be

\[
T_{\text{infl}} = \frac{d\Delta L}{dt} \sim \dot{M} \Omega_s r_s r_{H,d}. \] (6)

Setting this inflow torque equal to the dynamical friction torque in eq. (1), we find the scale for the mass inflow rate to be

\[
\dot{M} \sim \frac{M_s}{t_{\text{diff}}} \left( \frac{M_{\text{enc},s}}{M_d} \right)^{1/3}, \] (7)

where we have used the definition of the Hill radius of the ring given in eq. (4). Had we instead (naively) set eq. (4) to be equal to eq. (1), we would have found \( \dot{M} \sim M_s / t_{\text{diff}} \), which can be significantly smaller than the more physical scale in eq. (7).

As shown in § 2.1 above, \( t_{\text{diff}} \approx t_{\text{cross}} / q' \), so that \( \dot{M} \sim (M_d / t_{\text{cross}}) q'^2 / q_d^{1/3} \). Substituting the average (gas) mass inside the ring of \( M_d \sim 5 \times 10^7 M_\odot \), the crossing time of \( t_{\text{cross}} \sim 20 \text{Myr} \), and the observed range in ring-to-enclosed mass of \( q_d = 0.01 - 0.03 \), the typical scale for the mass inflow rate becomes \( \dot{M} \sim 10 q'^2 M_\odot \text{yr}^{-1} \). For star clusters with masses that are 1% (\( q' = 0.01 \)) to 10% (\( q' = 0.1 \)) of the nuclear ring mass, this implies a scale \( \dot{M} \) from about 0.001 to 0.1 \( M_\odot \text{yr}^{-1} \). To balance the combined dynamical friction from the many star clusters that are formed, the total mass inflow rate might easily have to be as high as \( \sim 1 M_\odot \text{yr}^{-1} \), which is the typical star formation rate in nuclear rings (e.g. Mazzuca et al. 2008). For mass inflow rates significantly below this scale, we expect the effect of this source of (gas) mass to be insignificant. For mass inflow rates at or above this scale, the effect of dynamical friction to push the cluster-ring system as a whole to smaller radii is halted.

2.3. Basic Equations

We now present the governing equations for our model. We do not present a detailed and complete derivation because these equations have been already derived in much of the literature (e.g. Lin & Papaloizou 1979a,b, 1986; Hourigan & Ward 1984; Ward & Hourigan 1989; Rafikov 2002), with two differences. First, we include the effect of dynamical friction of the background bulge stars, which is extensively discussed in C08 (to which we refer the interested reader). Second, we will include a mass source term for, which models the inflow of gas mass transitioning from the \( X_1 \) to \( X_2 \) orbits.

The evolution of a \textit{viscous} gas ring under the influence of an external torque is given by equations for continuity and angular momentum. The equation of continuity is

\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial (rv_r \Sigma)}{\partial r} = S(r,t), \] (8)

where \( r \) is the radial coordinate, \( v_r \) is the radial component of the velocity, \( \Sigma \) is the surface mass density of the
gas in the ring, and $S(r, t)$ is a position and time dependent source term. The angular momentum equation is (Pringle 1981; Rafikov 2002)

$$\frac{\partial (\Sigma \Omega^2)}{\partial t} + \frac{1}{r} \frac{\partial (r v_r \Sigma \Omega^2)}{\partial r} = - \frac{1}{2 \pi r} \left( \frac{\partial T_{\text{visc}}}{\partial r} - \frac{\partial T_{\text{disk}}}{\partial r} \right) + S(r, t) \Omega^2$$

where $T_{\text{visc}} = -2 \pi r^3 \nu \Sigma \partial \Omega / \partial r$ is the viscous torque with $\nu$ the viscosity. Taking into account that $\Omega$ (through $M_{\text{enc}}$) is not an explicit function of time, we combine eq. (8) and eq. (9) to eliminate $v_r$, so that we are left with

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2 \pi r \Omega} \left[ \frac{\partial (r^2 \Omega)}{\partial r} \right]^{-1} \left( \frac{\partial T_{\text{visc}}}{\partial r} - \frac{\partial T_{\text{disk}}}{\partial r} \right) + S(r, t).$$

The radial motion of the star cluster is governed by the competing torques of dynamical friction and ring tides. Henceforth, the change of angular momentum of the star cluster with time is

$$M_s \frac{\partial (\Omega_s r_s^2)}{\partial t} = T_{\text{df}} - T_{\text{disk}}.$$

For the (bulge of the) spiral galaxy in which star cluster and nuclear ring are embedded, we adopt a density profile $\rho \propto r^{-2}$ (see § 4.1 for a discussion on the validity of such an isothermal profile). The corresponding mass profile is given by $M_{\text{enc}} = M_{\text{enc},0} (r/\rho_0)$, with $M_{\text{enc},0}$ the mass enclosed within radius $\rho_0$. Such a mass profile has the nice property that the orbital velocity, $v_{\text{orb}} = \sqrt{\Omega r}$, is constant, and thus $\partial(\Sigma r^2)/\partial t = v_{\text{orb}}^2 \partial \Sigma / \partial r$ in eq. (11). With this mass profile, the partial derivatives of the viscous and tidal torques reduce to (see also C08)

$$\frac{\partial T_{\text{visc}}}{\partial r} = 2 \pi v_{\text{orb}} \frac{\partial (r v \Sigma)}{\partial r},$$

$$\frac{\partial T_{\text{disk}}}{\partial r} \simeq \beta \frac{G M_{\text{enc}}^2 \rho_0}{r^3} \frac{r^3 \Sigma}{M_{\text{enc},0}} \frac{1}{(r - \rho)^3}.$$

For the viscosity we follow C08, adopting $\nu = \nu_0 (\Sigma / \Sigma_0)^2$ as suggested by Lin & Papaloizou (1986) and $\nu_0 = \alpha (Q \rho_0^2)^{1/2} / \rho_0 v_{\text{orb}}$ using the standard Shakura & Sunyaev (1973) α-disk with Toomre (1964) $Q_0$ parameter, and, as before, $q_d = M_d / M_{\text{enc},0}$ the ratio of the nuclear ring (gas) mass to the enclosed mass.

Substituting these expressions and rescaling the surface mass density and time respectively as $\sigma = \Sigma / \Sigma_0$ and $t' = t / t_{\text{df},0}$, with $t_{\text{df},0} = M_{\text{enc},0} / (\Omega_0 M_{\text{H},0} \ln \Lambda)$, equations (10) and (11) become

$$\frac{\partial \sigma}{\partial t'} = \alpha' \frac{q_d^2 \rho_0^2}{r} \frac{\partial^2 (r^3)}{\partial r^2} + \beta' q_d \frac{r^3 \rho_0}{r} \frac{\partial}{\partial r} \frac{r^3 \sigma}{(r - \rho)^3} + \gamma' q_d^4 \beta' \frac{1}{q_d^{1/3}}$$

$$\frac{\partial r_s}{\partial t'} = \beta' 2 q_d r_{\text{df},0} \int_0^{r_s} \frac{r^3 \rho_0}{(r - \rho)^3} dr - \frac{r_d^2}{r_s}.$$

As before, $q' = M_s / M_d$ is the ratio of the cluster mass to the nuclear ring (gas) mass, $\alpha' = \alpha Q \rho_0^2 / (2 \pi \Lambda) \ln \Lambda$ and $\beta' = \beta / (2 \pi \ln \Lambda)$ set the strength of the ring viscous and tidal torques relative to the dynamical friction torque, and the constant $\gamma'$ controls the amount of inflow torque due to gas coming in along $X_1$ orbits that transitions to $X_2$ orbits. The dimensionless function $f(r)$ defines the radial extent where this gas is deposited in the nuclear ring. We choose the form

$$f(r) = \begin{cases} \frac{1}{q'} & \text{if } r_{d,0}^2 - (\Delta r < r < r_{d,0}), \\ 0 & \text{otherwise} \end{cases}$$

so that for $\gamma = 1$, the inflow torque and dynamical friction torque approximately cancel, corresponding to the scale of the mass inflow rate in eq. (7).

Equations (11) and (15) are the governing equations governing our model. They are the same as respective equations (23) and (24) in C08, except for the extra source term at the right-hand-side of eq. (11). In addition, our initial conditions and timescales of interest are different from the scenario studied in C08.

3. RESULTS

We first estimate the rate at which a star cluster moves out from the nuclear ring. Next, we numerically solve the equations describing our model, without any additional source of gas, and in case of a constant mass inflow rate.

3.1. Estimate of the Rate of Cluster-Ring Separation

We give an order of magnitude estimate for the rate at which a newly formed star cluster will separate itself from the nuclear ring. For simplicity, we suppose that the gas in the nuclear ring does not react significantly to the tidal torque, i.e., $\Sigma \approx \Sigma_0 (\sigma \approx 1)$ and $r_d \approx r_{d,0}$ remain approximately constant. In this case, eq. (11) can be neglected, and ignoring the last term in eq. (11) due to dynamical friction, it simplifies to

$$\frac{\partial r_s}{\partial t'} \sim 2 \beta' q_d r_{\text{df},0} \int_0^{r_s} \frac{r^3}{(r - \rho)^4} dr.$$

Since $r_s - r < \rho$, the latter integral out to the edge of the ring $r_{\text{df},0}$ can be approximated as $[r_{\text{df},0} / (r_s - r_{\text{df},0})]^4 / 3$. This leads to an ordinary differential equation in $x = (r_s - r_{\text{df},0}) / r_{\text{df},0}$ of the form $dx / d't' = (2 \beta' q_d / 3) x^{-3}$. The resulting solution, together with the Hill radius $r_{H,d}$ defined in eq. (6) and $r_s \approx r_{d,0} \gg r_{H,d}$, implies that

$$\left( \frac{r_s - r_{d,0}}{r_{H,d}} \right)^4 \sim \frac{8 \beta' q_d}{3} \frac{t}{r_{H,d} t_{\text{df},0}}$$

Since the left-hand-side as well as $8 \beta' / 3$ are of order unity, the timescale for a star cluster to separate itself from the nuclear ring in which it was formed is of order $t_{\text{sep}} \sim (r_{H,d} / r_{\text{df},0}) t_{\text{df},0}$. The observed range of $q_d = 0.01 - 0.03$ implies $t_{\text{sep}} / t_{\text{df},0} \sim 0.2 - 0.3$. Furthermore, in units of the crossing time $t_{\text{cross}} = t_{\text{df},0} q_d^2 r_{2\text{df},0} / r_{H,d}$, the separation time is about $t_{\text{sep}} / t_{\text{cross}} \sim (r_{H,d} / r_{\text{df},0}) / q_d$. This means a few to tens of orbits for cluster-to-ring mass ratios from $q' = 0.01$ to $q' = 0.1$, or about 500 Myr to 50 Myr for a typical crossing time of $t_{\text{cross}} = 20$ Myr. Since the time $t$ in eq. (18) goes as a fourth power of the cluster-ring separation $r_s - r_{d,0}$, the initial separation is much faster than $t_{\text{sep}}$. This means that for a star cluster to reach a significant fraction of the Hill radius of the nuclear ring is very fast, i.e., typically of order less than 1% of the dynamical friction time. This implies a factor 20 - 30 quicker than $t_{\text{sep}}$, or, with $t_{\text{cross}} = 20$ Myr, less than 20 Myr for $q' = 0.01$ and less than 2 Myr for $q' = 0.1$. We come back to this separation in the next
torque will be very strong and pushes the star cluster a realistic system. Nevertheless, the initial ring tidal of the tidal torque in eq. (13), and may not develop in enhancement is due to the fourth-power radial dependence of the initial dynamical friction time, \( t_{\text{df}} \), as function of the time, \( t \), in units of the initial dynamical friction time, \( t_{\text{df,0}} \). The bottom panel shows snapshots of the surface density profile at the edge of the nuclear ring for various times, separated by \( \Delta t/t_{\text{df,0}} = 0.06 \), with labels A-D as indicated in the top panel. The star cluster has mass of 1% \( (q' = 0.01) \) relative to the gas mass in the nuclear ring. Furthermore, the dimensionless model parameters are set to \( \alpha' = 0.01, \beta' = 1 \), and \( \gamma' = 0 \) (no mass inflow).

3.2. Separation and Density Enhancement

We solve equations (14) and (15) using standard explicit finite-difference methods (Press et al. 1992). For good spatial resolution, we use 2000 grid points in \( r/r_{d,0} \) between zero and 1.50, with the star cluster initially at \( r_s/r_{d,0} = 1.01 \). We set \( q_d = M_d/M_{\text{enc}} = 0.01 \) similar to the observed ratios of (gas) mass in the ring to enclosed mass within the ring radius derived in \( \S \text{2.1} \). Furthermore, for the dimensionless parameters \( \alpha' \) and \( \beta' \), representing the strength of the ring viscosity and tides relative to the dynamical friction, we choose \( \beta' = 1 \) and \( \alpha' = 0.01 \). We first study the case without source term, i.e., \( \gamma' = 0 \), while in the next section \( \S \text{3.3} \) we include the effect of gas mass inflow.

In Figure 2, we show the (numerical) solution of equations (14) and (15) for a star cluster with mass relative to the (gas) mass in the nuclear ring of \( q' = 0.01 \). The top panel shows the evolution of the radial position of the star cluster, \( r_s \), relative to the initial outer radius of the nuclear ring, \( r_{d,0} \), as function of the time, \( t \), in units of the initial dynamical friction time, \( t_{\text{df,0}} \). The bottom panel shows snapshots of the surface density profile at the edge of the nuclear ring for various times, separated by \( \Delta t/t_{\text{df,0}} = 0.06 \), with labels A-D as indicated in the top panel. The initial built-up of material at the edge of the ring is very fast and only after \( t/t_{\text{df,0}} = 10^{-4} \) (point A) an enhancement of about 30% in the surface mass density is achieved just inside \( r_{d,0} \). This rapid initial enhancement is due to the fourth-power radial dependence of the tidal torque in eq. (13), and may not develop in a realistic system. Nevertheless, the initial ring tidal torque will be very strong and pushes the star cluster outward beyond the initial edge of the ring. After a time \( t/t_{\text{df,0}} = 0.06 \) (point B), the star cluster has moved out about 20% from its initial position. The surface mass density near the outer edge decreases now with respect to its initial value due to viscous spreading. Points C and D continue the trend of an outward moving star cluster and viscously spreading ring, albeit at a decreasing pace.

In contrast, for the case of \( q' = 0.1 \) (a ten times more massive star cluster), the star cluster does not continuously move outward, but rather stop and even moves inward as shown in Figure 3. As in the earlier case, an initial surface mass density enhancement is built at point A, though it is a factor of ten larger than for the case \( q' = 0.01 \), i.e., 300% \( (q' = 0.1) \) as opposed to 30% \( (q' = 0.01) \) of its initial value. However, the subsequent evolution of this case differs from the earlier case. The star cluster still moves outward, but it reaches only about 15% of its initial position. Its maximum in \( r_s/r_{d,0} \) is reached between point B and C, after which the star cluster “turns around” and starts moving inward with respect to the initial outer radius of the ring. The surface mass density at the edge of the ring monotonically moves inward, effectively being “shepherd” inward by the star cluster, similar to the scenario in C08.

In both cases, the radial separation between the star cluster and the (edge of the) nuclear ring approach a constant value, which we expect from the order of magnitude estimate in \( \S \text{2.2} \) to be close to the Hill radius of the ring, \( r_{H,d} \), as defined in eq. (4). Substituting \( q_d = 0.01 \) and using that \( r_s \geq r_{d,0} \Rightarrow r_{H,d} \), we estimate this Hill radius to be around one-fifth of the initial radius of the ring, i.e., \( r_{H,d}/r_{d,0} \sim 0.2 \). Combining the above shifts in the radial positions of the star cluster and nuclear ring, we find that the separations for both cases indeed converge to around one-fifth of \( r_{d,0} \), although for \( q' = 0.01 \) the separation stays slightly smaller, while for \( q' = 0.1 \) it is somewhat larger. This is further illustrated in Figure 4 where we show the evolution of the radial separation \( r_s - r_{d,\text{edge}} \) normalized by the Hill radius \( r_{H,d} \).
use two definitions of the radius of the disk edge, \( r_{d,\text{edge}} \). For \( q' = 0.01 \), we define \( r_{d,\text{edge}} \) as largest \( r_d \) for which \( \Sigma/\Sigma_0 > 75\% \), while for \( q' = 0.1 \), \( r_{d,\text{edge}} \) is defined as the \( r_d \) for which \( \Sigma \) peaks. Despite these two different definitions, Figure 4 shows two important points. First, the separation between the disk and satellite is of order \( 10 \) times more massive star cluster (\( \times 10^3 \)), normalized by the Hill radius \( r_{H,d} \) of the nuclear ring. The top curve is for a ten times more massive star cluster (\( q' = 0.1 \), see Figure 3) then bottom curve (\( q' = 0.01 \), see Figure 2). For both cases, the time to converge to the final separation is about 20% of the initial dynamical friction time \( t_{df,0} \), while the initial separation is very fast, reaching half of the final separation in less than 1% of \( t_{df,0} \).

The time evolution of the radial separation \( r_s - r_{d,\text{edge}} \) between the star cluster and the edge of the nuclear ring, normalized by the Hill radius \( r_{H,d} \) of the nuclear ring. The top curve is for a ten times more massive star cluster (\( q' = 0.1 \), see Figure 3) then bottom curve (\( q' = 0.01 \), see Figure 2). For both cases, the time to converge to the final separation is about 20% of the initial dynamical friction time \( t_{df,0} \), while the initial separation is very fast, reaching half of the final separation in less than 1% of \( t_{df,0} \).

3.3. Constant Mass Inflow Rate

We now include the effect of a constant gas mass inflow on the dynamics of the cluster-ring systems. Figure 5 shows the results of (numerically) solving the same system with \( q' = 0.1 \) as in Figure 3 except that the source term in eq. (1) now contributes with \( \gamma' = 1 \). We estimated in § 2.2 that the latter corresponds to an mass inflow rate of \( \dot{M} \approx 0.1 M_\odot \text{yr}^{-1} \). Comparing the top panels of the two figures, we find that the star cluster moves (slightly) further out when \( \gamma' = 1 \) and remains constant at the maximum radial position, as opposed to the shrinking of \( r_s \) after having moved out for an initial period when \( \gamma' = 0 \). This is because in the case that \( \gamma' = 1 \), the inflow torque is balancing the torque due to dynamical friction, as we estimated in eq. (7). The bottom panels of the two figures are very similar, both showing an enhancement as well as inward migration of the surface mass density. The most significant difference is that in Figure 5, beyond the edge at \( r/r_{d,0} \sim 0.92 \) there is a long tail out to \( r/r_{d,0} \sim 1 \), due to mass inflow into this region. Another difference is that for \( \gamma' = 1 \) after the initial inward push of the edge, it remains fixed at \( r/r_{d,0} \sim 0.92 \), whereas for \( \gamma' = 0 \) it still moves inward, although at a decreasing rate. This is consistent with the above difference in the change in the radial position \( r_s \) of the star cluster. As a consequence, also in the case that \( \gamma' = 1 \), the separation between the star cluster and the (edge of the) nuclear ring converges to a constant value, which is again of the order of the Hill radius of the ring.

In Figure 5, we show the evolution of the radial position of the star cluster relative to the initial outer edge of the ring \( r_s - r_{d,0} \), in units of the latter Hill radius \( r_{H,d} \), for four different constant mass inflow rates. From bottom to top the curves correspond to \( \gamma' = 0 \), 0.3, 1 and 3. This clearly shows the transition around \( \gamma' = 1 \) after a similar initial outward migration, the motion of the star cluster goes from in-falling for \( \gamma' \lesssim 1 \) to outgoing for \( \gamma' \gtrsim 1 \). This is fully consistent with our estimated scale for the mass inflow rate \( \dot{M} \) in eq. (7), case of single star cluster with \( q' = 0.1 \).

4. COMPARISON TO OBSERVATIONS

We describe the expected evolution of nuclear rings due to interaction with star clusters based on our findings in the previous sections. First, we focus on the evolution in radius, and discuss possible azimuthal variations thereafter. Finally, we take a more detailed look at the well-studied nuclear ring in NGC 4314.

4.1. Radial Evolution of Nuclear Rings

From our analysis in § 2.2 we expect that a newly formed cluster moves outward until it is about a Hill radius separated from (the edge) of the nuclear ring. We derived in § 2.2 a range in \( r_{H,d} \) from about 50 pc to 500 pc, with 150 pc as a typical value. For a host galaxy at distance of \( \sim 10 \text{ Mpc} \), this corresponds to about \( 1' \) to \( 10'' \), and typically \( 3'' \), which should be readily observable. However, while the position of the (young) star clusters with respect to the center of their host galaxy is straightforward to measure, the radial extent of the gas in the...
ring is often difficult to determine. It requires high spatial resolution CO measurements to trace the underlying molecular gas. Also, part of the gas (at the edge) of the ring may be severely disturbed due to winds that clear out the gas and dust from the newly formed star clusters and/or ionized by the massive, bright stars in the young star cluster (but see § 5.3 and § 5.4). Still, the dust seems to be well mixed into the gas (e.g. Brunner et al. 2008), indicating that dust provides a good tracer of the extent of the gas in the ring. Apart from the contact points where the bar dust lanes connect with the nuclear ring, the dust is indeed found on the inside of the distribution of the star clusters (see also § 2.2).

Still, using the (peak in the) dust distribution as an estimate of the edge of the ring, the (unique) measurement of a separation is complicated by the spread in radius of the star clusters. Part of this spread is because of the dependence of the dynamical friction time on the mass of the star cluster: \( t_{df}/t_{cross} \sim 1/q' \). Even though we found that it takes less than 1% of the dynamical friction time to reach about half of the Hill radius, with a typical crossing time of \( t_{cross} = 20 \) Myr, this means only 2 Myr for the most massive star clusters \( (q' = 0.1) \), but as much as 200 Myr for the least massive clusters \( (q' = 0.001) \). Since \( t_{sep}/t_{df} \sim 0.2 - 0.3 \), to reach the full separation takes a factor 20 – 30 longer. This means that only the most massive clusters reach at least half of the Hill radius separation before all bright, massive stars die. The less massive clusters have already faded significantly, or even dissolved. So, we then might expect a smooth radial gradient with the most massive clusters on the outside, which in turn can be used to establish the Hill radius.

Unfortunately, we found that the more massive cluster indeed rapidly reach a maximum separation, but after staying a while at this radius (depending on the mass inflow rate), they move inward again by pushing the ring inward (compare for example the top panels of Figures 2 and 3). Hence, over time the less massive clusters, if they survive, may catch up with the more massive clusters. We conclude that only among the (very) young (few Myr) clusters we might expect a radial gradient with the more massive clusters to the outside, from which the Hill radius can be estimated. Later on, if a radial gradient is visible at all, it might even be opposite, with the more massive clusters on the inside. Since the more massive clusters have higher survival chance, the latter ‘turn around’ may result in at most a mild radial gradient in age (with the older clusters on the inside).

**4.2. Azimuthal Age Gradient**

Unlike the radial age gradient, Mazza et al. (2008), do find signatures of an azimuthal age gradient in most of the nuclear rings, ranging from clean bipolar age gradients to a correlation between the youngest star clusters and the contact points of the bar dust arms with the nuclear ring. The natural explanation is that at (or very close to) these contact points the inflowing gas shocks and forms stars, after which the resulting star cluster travels in the ring away from the contact points. Indeed, for a typical bar pattern speed of \( \sim 50 \) km s\(^{-1}\) kpc\(^{-1}\) (e.g. Gerssen 2002) the corresponding time for the contacts points to rotate is \( \sim 120 \) Myr, significantly shorter than the local crossing time \( t_{cross} \sim 20 \) Myr. Alternatively, if the star formation occurs due to (local) gravitational instability of the gas in the ring, the star clusters are expected to form throughout the ring without an azimuthal age gradient. Instead, of associating this instability with an inner Lindblad resonance (ILR; e.g. Elmegreen 1994), it is a natural consequence of shepherding: The gas in the ring loses angular momentum to the star cluster(s), which in turn lose angular momentum to the host galaxy through dynamical friction. This means that gas at the edge of the ring moves inward, resulting in an increase in the gas surface density at the edge (see also Figures 2 and 5), until its becomes unstable against self-gravity.

In the case of shepherding we do expect, as observed, a mixture in azimuthal age gradients. Initially, when the gas surface density in the ring is below the critical value for star formation, star clusters only form in episodes at the two contact points, they move together with the remaining gas along the ring, resulting in bipolar age gradient. The newly formed star clusters move outward, while at the same time ‘pushing’ the gas inward, causing an over-density built-up at the edge of the ring. When the gas surface density somewhere along the edge of the ring reaches the critical value, it becomes unstable and forms stars. The more massive the star cluster (or group of star clusters acting together), the higher the over-density and the sooner the critical value is reached.

Generally, the most massive clusters and hence the star formation might be anywhere along the ring. However, depending on the mass inflow rate, there might be relatively more gas near the contact points to start with, so that first instability-driven star clusters still form near the contact points. Therefore, an azimuthal age gradient might still exist over part of the ring, but eventually the gradient is expected to flatten.
We found in §4.3 that for a low enough mass inflow rate, the (most massive of the) existing star clusters move inward by dynamical friction, while 'pushing' and increasing the surface mass density of the gas in the outer part of the nuclear ring. As a result, the gas might reach the critical surface mass density and form new stars (in clusters) all along the edge of the ring. At the same time, near the contact points, which also move inward along with the nuclear ring, star formation from infalling and shocking gas might be ongoing. We argue below in §4.3 that the nuclear ring in NGC 4314 is an example of this mechanism. Since even in the case of a high mass inflow rate, eventually enough (massive) star clusters are formed for their (combined) dynamical friction to 'push' to ring inward, the corresponding gas and bright star clusters migrate inward from larger to smaller $X_2$ orbits.

In other words, nuclear rings will be located between the so-called outer and inner ILR, although strictly speaking the predicted Hill radius, which is expected based on the mass in the ring from $\sim 1.0''$ to $1.8''$, bracketing the observed difference.

There is no evidence for an azimuthal age gradient, but a slight tendency for a radial age gradient with the few clusters outside the ring begin all older than 15 Myr. This is consistent with the shepherding scenario in which the gas in the ring has been 'pushed' inward until a critical surface density is reached and all along the edge of the nuclear ring star clusters are formed, which then move outward. The (group of) star cluster(s) responsible for the inward migration of the ring likely have faded too much to be visible against the old underlying stellar population of the host galaxy. However, further support for the inward migration of the ring comes from two blue stellar spiral arms that seems to connect the outer-most $X_2$ orbits (also referred to as the outer ILR) with the nuclear ring [Wozniak et al. 1995; Benedict et al. 2002]. Subtracting a model of the red host galaxy, Benedict et al. 2002 find that the stars in the arm close to the nuclear ring are $\sim 20$ Myr old, with an indication of an increase in age beyond $\sim 100$ Myr going outward in the arms. Whereas during the inward migration the gas surface density along (the edge of) the ring was insufficient to form stars, shocks at the contact points might still have allowed the formation of stars. While the contact points moved inward together with the ring, these stars stayed behind and created the blue arms.

Benedict et al. (1996) derive an outer ILR at $\sim 13''$ and inner ILR at $\sim 4''$ for an assumed bar pattern speed $\Omega_p \sim 72$ km s$^{-1}$ kpc$^{-1}$ of the large-scale bar. The latter is likely an upper limit because typically the co-rotation radius is beyond the $\sim 3$ kpc radius of the bar, so that with a $190$ km s$^{-1}$ circular velocity, $\Omega_p \lesssim 60$ km s$^{-1}$ kpc$^{-1}$. In turn this implies that the above outer and inner ILR are probably lower and upper limits respectively. This clearly shows that it is critical to obtain an independent measurement of the bar pattern speed, using for example the Tremaine-Weinberg (1984) method. Even so, the nuclear ring is in between the outer and inner ILR, in line with the shepherding scenario, in which the ring migrates inward until the increasing gas surface density at the edge reaches the critical value for star formation. While the newly formed star clusters move (slightly) outward peaking at $\sim 7.0''$, the gas in the ring is 'pushed' further inward peaking at $\sim 5.5''$. Note that Benedict et al. (1993) detected a nuclear bar within $\sim 4''$ that might provide a mechanism to transport part of the gas further inward towards the presumed black hole in the center (see also Benedict et al. 2002).

5. DISCUSSION

We discuss some of the assumptions of our model and speculate about possible consequences the inward migration of the cluster-ring system has on transporting gas to the center of the galaxy.

5.1. Isothermal Density Profile

In deriving the equations for our model in §4.3, we have adopted a density profile $\rho \propto r^{-2}$ for the (bulge of the) spiral galaxy in which the star cluster and nuclear ring are embedded. At the radii where nuclear rings are present (0.2–1.7 kpc, Mazzuca et al. 2008), the density of galaxies indeed seem to be not too far
from such an isothermal profile, as indicated, for example, by studies that combine stellar dynamics with strong gravitational lensing (e.g. Koopmans et al. 2006; van de Ven et al. 2008). A more straightforward way is to consider that the surface brightness profiles of (bulges of) galaxies are well fitted by a Sésérid (1968) profile \( I(R) \propto \exp[-(R/R_e)^{1/n}] \), with index \( n \) and the effective radius \( R_e \) enclosing half of the total light. The (numerically) deprojection is well approximated by the density profile of Prugniel & Simien (1997), of which the (negative) slope is given by

\[
\gamma = \frac{n}{n-1} \left( \frac{r}{R_e} \right)^{1/n},
\]

with \( p_n = 1.0 - 0.6097/n + 0.05463/n^2 \) and \( b_n = 2n - 1/3 + 4/405n + 46/25515n^2 \) (Ciotti & Bertin 1999). For an average bulge of a spiral galaxy \( n \sim 2 \) and \( R_e \sim 1 \) kpc (e.g. Hunt et al. 2004), so that at the typical radius \( r \sim 0.5 \) kpc of a nuclear ring, we indeed find that the slope of the density \( \gamma \sim 2 \) is close to isothermal.

### 5.2. Gas Clumps and Clouds

The star cluster that is being formed in the nuclear ring is already subject to dynamical friction while it is still a gas clump, as long as the material that makes up the (proto) star cluster is bound enough relative to the background stellar population of the host galaxy. Since this gas clump contains (significant) more mass than the final cluster of only stars, the initial dynamical friction is (much) stronger. However, the clearing of gas (and dust) from the newly formed star cluster is very fast and efficient, since even the youngest visible star clusters are only mildly reddened (e.g. Maoz et al. 2001).

We assume that the gas in the nuclear ring is uniform, i.e., no gas clouds (or clumps) aside from those which turn into star clusters. Since such gas clouds would also be subject to dynamical friction from the background bulge stars and thus move inward, dynamical friction on the nuclear ring itself has also been proposed as an explanation of the inward migration of the nuclear ring in for example NGC 4314 (Combes et al. 1992).

Suppose that the gas is made up of \( N \) gas clouds of the same mass \( M_g = M_d/N \), then about \( N = 1/\sqrt{2} \) clouds are needed to have a combined dynamical friction torque of the same order as that of a single star cluster of mass \( M_s = q'M_d \). Adopting as before the two cases of \( q' = 0.01 \) and \( q' = 0.1 \), this respectively means \( N = 10^4 \) and \( N = 10^2 \) gas clouds of mass \( M_g = 5 \times 10^3 M_\odot \) and \( M_g = 5 \times 10^5 M_\odot \) for a typical nuclear ring mass of \( M_d = 5 \times 10^7 M_\odot \). From \( \S 2.4 \) we have that the dynamical friction time in units of the crossing time is approximately given by \( t_{df}/t_{cross} \approx M_d/M_g \), so that with \( M_s = M_e = M_d/N \) for a gas cloud we obtain \( t_{df} \approx N t_{cross} \). Given a typical crossing time of \( t_{cross} \sim 20 \) Myr, the timescales of dynamical friction for the latter two cases are about \( t_{df} \sim 200 \) Gyr and \( t_{df} \sim 2 \) Gyr, respectively. We thus conclude either nearly no inward migration \( (q' \sim 0.01) \), or gas that is very clumpy with quite large gas clouds \( (q' \sim 0.1) \).

Moreover, since the gas clouds will span a range in mass with different corresponding dynamical friction timescales, they migrate inward at different rates, leading to a (radial) diffusion of the nuclear ring, especially with respect to the gas that is not in clouds and stays behind. Although in some galaxies, like M 83, that are claimed to harbor a (partial) nuclear ring, the corresponding morphology is rather clumpy and chaotic, most of the nuclear rings are are quite smooth and well defined, including the nuclear ring in NGC 4314. Collisionless star clusters are also less prone to destruction than collisional gas clouds, in particular in the presence of shear stresses from differential speeds. As a result, at least the most massive star clusters will sink faster inward than the gas clouds, and appear to be on the inside of the gas (clouds) in the nuclear ring, which typically is not observed.

### 5.3. Star Formation within the Nuclear Ring

We have assumed that the star cluster form just exterior of the nuclear ring. This assumption simplifies the physics and the numerical computation, though it is not absolutely necessary. If the star cluster forms near the outer edge of the nuclear ring, we do not expect the resulting dynamics to be significantly different. By near, we demand that the separation between the star cluster and the outer edge is within one Hill radius of the nuclear ring. In this way, the amount of mass which participates in the tidal evolution of the cluster-ring system exterior to the star cluster is less then interior to the star cluster. As long as this condition is fulfilled, the evolution should proceed along the lines of our calculation.

There are a few reasons to believe that the star clusters (preferably) form near the outer edge of the nuclear ring. The supply of gas from the bar merges with the ring at the outermost radius. It is unknown how the gas spreads radially over the rest of the ring, but if it is due to viscous processes, for instance the magneto radial instability (MRI; Balbus & Hawley 1991), then the surface density is maximal at the outer edge. As a result, the gas in the ring is most likely to be (come) unstable near the outer edge and hence preferentially fragment there. While many clusters will form near the outer edge (and hence susceptible to shepherding) for the reasons given above, clusters are also likely to form inside of the outer edge. For instance, once massive star clusters are formed, their interaction with the gas ring may result in large and fairly symmetric enhancements in the gas surface mass density (see for example Figure 3), so that new, but significant smaller, star clusters can be formed around these enhancements. For the relatively small star clusters formed on the inside, we expect the inward migration to follow that of forming proto planets (see e.g. Ward 1997). In addition, if the gas flowing along the bar is very clumpy, possess a different vertical scale height, or a different inclination than the gas ring, it may not (only) merge at the outermost radius. In this way, even rather large star clusters might form on the inner edge of the nuclear ring. They then migrate further inward (relatively faster than cluster formed on the outer edge as they encounter less of the gas ring), possibly along an inward extension of the inspiraling gas. Whereas these are possible explanations for the observation that some star clusters appear more on the inside of the nuclear ring (see also §2.2), alternatively they might form already inside of the ring as the result of the local interaction of a non-axisymmetric structure such as a nuclear bar or spiral with the interstellar medium (see also §5.5).
5.4. Stellar Feedback

Winds originating from (massive) stars and/or supernovae in the star clusters might not only clear out the remaining gas in the star cluster itself, but also push away part of the gas in the nuclear ring. This creates a separation between the cluster and the ring that might mimic the outward migration of the star cluster. However, since the density of the interstellar medium is far less than that of the gas in the ring, the winds will escape in the vertical direction like a fountain arising from the (equatorial) plane of the ring. Therefore, even if the cluster is embedded in the ring itself, the winds are expected to clear out at most a region that is equal to scale height of the disk, $h_d$, which is significantly smaller than the Hill radius of the ring, $r_{H,d}$.

To show this, we start from Toomre (1964) $Q$ parameter for the gas in the ring: $Q = \frac{c_s}{\kappa} \frac{\Omega}{\sqrt{\Sigma}}$ (Binney & Tremaine 1987). Here, the epicycle frequency $\kappa$ is defined as $\kappa^2 = (4 + d \ln \Omega/\ln r) \Omega^2$, so that $\kappa = \sqrt{\frac{2}{3}} \Omega$ for the assumed (isothermal) density profile $\rho \propto r^{-2}$. Adopting $c_s = h_d \Omega$, we obtain for the scale height of the ring $h_d/r_d \simeq (M_d/M_{enc}) Q/\sqrt{2}$. On the other hand, the Hill radius of the ring $r_{H,d}/r_d \simeq (M_d/M_{enc})^{1/3}$. Assuming $Q \simeq 1$ and substituting the mean observed ring-to-encoded mass ratio $M_d/M_{enc} \simeq 0.02$, we find that $h_d/r_{H,d} \simeq 0.05$. This indeed shows that winds might clear out a region that is at most a few per cent of the Hill radius of the nuclear ring.

A second possible stellar feedback effect, is due to the (ultraviolet) radiation coming from the massive, bright stars in the ring (molecular) gas in the nuclear ring. We can estimate the radial extent of this ionization from the radius $r_{ion}$ of the Strömgren sphere. We start from $N_{ion} \simeq \alpha n_d^2 4 \pi r_{ion}^3 /3$, with cross section $\alpha \simeq 3 \times 10^{-13} \text{ cm s}^{-1}$. We assume that bright stars radiate roughly at 10% of the Eddington luminosity in the ultraviolet (UV; $\geq 10^4 \text{ eV}$), to obtain an ionization rate of $N < 10^{48} (M_d/M_\odot)$ photons per second. With the above estimate of the scale height $h_d$, we find for the gas mass density $\rho_g \simeq M_{enc} \sqrt{2/r_d^3} Q$. Substituting $M_{enc} \sim 2.5 \times 10^6 M_\odot$, $r_d \sim 0.5$ kpc and $Q \simeq 1$, and dividing by the proton mass, we find for the number density of the gas $n_g \sim 4 \times 10^3 \text{ cm}^{-3}$. In this way, we estimate that a (UV) radiating star of mass $M_\star$ is able to ionize gas out to a (maximum) radius of $r_{ion} < 3 (M_d/M_\odot)^{1/3} \text{ pc}$.

For a typical O star of $M_\star \sim 40 M_\odot$, this means $r_{ion} < 10 \text{ pc}$, which is significantly smaller than the typical Hill radius of $r_{H,d} \sim 150 \text{ pc}$. The combined (and focused) ionizing radiation of thousands of massive young stars with a lifetime of only a few to ten Myr are needed to reach the latter radius. Even then, the dust, which is hardly affected by the radiation, will still provide a good tracer of the gas in the ring, and as such can be used to establish the separation between star clusters and the nuclear ring.

5.5. Feeding the Nucleus?

For sufficiently massive star clusters and/or for a sufficiently large number of them, it is in principle possible for these star clusters to shepherd the nuclear ring to smaller radii. As in C08, we assumed the gas ring sits in a simple spherically symmetric potential. However, the gas ring arises as a result of the stellar bar, which is a non-axisymmetric feature in the potential, driving the gas inward towards the $X_1 - X_2$ orbit transition. In addition, the gas ring is self-gravitating with Toomre’s (1964) $Q$ near unity. We now speculate on the effect of this non-axisymmetry and self-gravity on the gas shepherd scenario.

First, as the gas ring is being shepherd inward, it moves from one $X_2$ orbit to a smaller one. Eventually, it may move inward of the smallest non-intersecting $X_2$ orbit in a barred potential (e.g. Regan & Teuben 2003). The gas which is pushed beyond this point will find itself in intersecting orbits, losing angular momentum on each orbit as it shocks against gas on the same orbit or on neighboring orbits. The high(er) density regions resulting from the shocks, might give rise to the observed (chaotic) spurs of gas and dust in the centers of late-type (elliptical) galaxies (e.g. Elmegreen et al. 1998; Martini et al. 2003).

Second, the effect of self-gravity in the shepherded gas ring is nontrivial. As we stated above in § 5.4, the effect of an enhanced surface density at the edge of the ring results in $Q \lesssim 1$, if the initial $Q \simeq 1$. In addition, this local enhancement in the surface density may give rise to the gaseous analogy of stellar self-gravitating groove and edge modes (see also Lovelace & Hohlfeld 1978; Sellwood & Kahn 1991; Papaloizou & Savonije 1991). This may result in the formation of nuclear bars (e.g. Laine et al. 2002; Erwin & Sparke 2002; Englmeier & Shlosman 2004) or nuclear spirals (e.g. Englmeier & Shlosman 2000; Posse & Martin 2002), providing a means for inward transport of gas from the nuclear ring region in analogy to the “bars within bars” scenario (Shlosman et al. 1994).

6. Conclusions

We have studied the dynamics between a star cluster and the gas in the nuclear ring from which it was formed. The star cluster is subject to dynamical friction with the background bulge stars in the host galaxy. The resulting dynamical friction torque induces a tidal (and viscous) torque on the gas in the nuclear ring. In addition, we investigated a torque due to in-falling gas along the bar onto the nuclear ring at the contact points. We arrived at the following, observationally testable predictions based on our model.

(i) The star clusters which are formed near the outer edge of the ring migrate outward. The final separation is of the order of the Hill radius of the ring, which is 20−30% of its radius if the ring’s gas mass is the observed 1−3% of the enclosed mass. Similarly, the time to reach this final separation is about 20−30% of the dynamical friction time. The initial separation is very fast, reaching half the Hill radius within only 1% of the dynamical friction time. The latter can be as short as a few million years for a massive enough ($\gtrsim 10^6 M_\odot$) star cluster.

(ii) While such a massive cluster moves out the surface mass density of the gas at the edge of the nuclear ring is pushed inward and gets enhanced by a factor of a few. If this enhancement reaches the critical value for star formation, new star clusters are
expected to form along the ring. This is in addition to the (ongoing) formation of star cluster from shocking gas near the contact points. The (initial) azimuthal age gradient that is expected from the latter gets diluted or even washed out by the former (secondary) star cluster formation.

(iii) If the (formed) star cluster are massive and/or numerous enough, and the gas mass inflow rate low enough (or even shut down), we expect the inward migration of the cluster-ring system as a whole. This mean that, even though initially the nuclear ring could have formed as often suggested at an (inner) Lindblad resonance, it can migrate closer to the center of the host galaxy.

The latter implies that the most massive star cluster, after an initial outward separation, turn around and migrate inward along with the nuclear ring. Consequently, any initial radial age gradient due to the dynamical friction time being inverse proportional to the mass of the star cluster, is diluted or even inverted. Even so, both a radial and azimuthal age gradient are observationally difficult to measure. When the most massive stars are still alive, the bright emission lines can be used for (relative) radial and azimuthal age gradient are observationally difficult to measure. Therefore, the dynamical friction time is expected to be of the order of the Hill radius of the most massive star cluster, which indeed is the case in NGC 4314. This well-studied nuclear ring also seems to have migrated inward, leaving a tail of older star clusters behind and being currently located between the inner and outer ILR. To firmly confirm such a “shepherding of gas” towards smaller radii, the dynamics of the host galaxy, including the (bar) pattern speed, needs to be measured accurately.

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REFERENCES

Buta, R. & Combes, F. 1996, Fundamentals of Cosmic Physics, 17, 95